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FUNDAMENTALS OF METHODOLOGICAL CONTROL OF THE STRENGTH OF COMPOSITE MATERIALS

The idea of this paper is to propose research methods designed for strength analysis of composite materials which are widely used in load-carrying structures. It is shown that the strength analysis and study of composite materials are carried out in two directions – considering them as inhomogeneous composite materials, representing a regular multilayer medium of alternating reinforcement layers and a polymer binder, and the study of the issue of structural strength, in which composites are considered as homogeneous elastic orthotropic bodies, to which the theory of elasticity of anisotropic media is applicable. These approaches enables to represent the composites in the form of a continuous medium and use the methods of the theory of elasticity of anisotropic media in calculating the acting stresses, which makes it possible to use the influence of the elements of manufacturing technology on the physical and mechanical characteristics.

Keywords: *composite materials (CM), strength, stiffness, elasticity*

Introduction

Modern composite materials certainly now constitute one of the most important classes of engineering materials widely used in critical load-carrying structural elements. There are several reasons for this. One is that they offer highly attractive combinations of strength, stiffness, lightness and corrosion resistance. At present, the study of the strength of composite materials is carried out in two directions.

In the works where the first direction is applied [9, 10] composite materials are considered as heterogeneous composites that represent a regular multi-layer medium of alternating reinforcement layers and a polymer binder.

When applying this theory, certainly, difficulties arise in connection with the presence of defects in the manufacture of structures, etc.

In the works where the second direction is used [4, 8], the structural strength study of homogeneous elastic orthotropic bodies, to which the theoretical foundations of the elasticity of anisotropic media are applicable, is becoming widespread. This assumption is based on the fact that the dimensions of the reinforcing filler are negligibly small compared with geometric dimensions of the part's cross-section. Based on this, the composites can be represented as a continuous homogeneous medium. This approach makes it possible to use simple and well-developed methods of the theory of elasticity of anisotropic media when calculating the acting stresses, which enables to take into account the influence of elements of manufacturing technology on the physical and mechanical characteristics of a structure. Orthotropic composite materials are most widely used in load-carrying structures, therefore, the main attention is paid to this issue.

Materials and Methods

The strength characteristics of quasi-homogeneous, nonisotropic materials are derived from a generalized distortional work criterion. For unidirectional composites, the strength is governed by the axial, transverse, and shear strengths, and the angle of fiber orientation. The strength of a laminated composite consisting of unidirectional layers depends on the strength, thickness, and orientation of each constituent layer and the temperature at which the laminate is cured. In the process of lamination, thermal and mechanical interactions are induced which affect the residual stress and the subsequent stress distribution under external load. A method of strength analysis of laminated composites is delineated using glass-epoxy composites as examples. The validity of the method is demonstrated by appropriate experiments.

Solution of the Problem

It is well known that in the theory of elasticity the stressed state of an anisotropic medium is described by the generalized Hooke's law

$$\sigma_{ix} = \mathrm{C}_{klm}\varepsilon_{im}; \tag{1}$$

where σ_{ix} are the components of the stress tensor; ε_{im} are strain tensor components; C_{klm} are components of the elastic modulus tensor (which is a fourth rank tensor). In this case, the elastic potential is a function of the second degree, invariant with respect to the coordinate degree, then $\mathrm{C}_{iklm} = \mathrm{C}_{ik}$.

For an orthotropic medium, Equation (1) in expanded form can be represented as

$$\left.\begin{aligned}
\sigma_x &= \mathrm{C}_{11}\varepsilon_x + \mathrm{C}_{12}\varepsilon_y + \mathrm{C}_{13}\varepsilon_z,\\
\sigma_y &= \mathrm{C}_{21}\varepsilon_x + \mathrm{C}_{22}\varepsilon_y + \mathrm{C}_{23}\varepsilon_z,\\
\sigma_z &= \mathrm{C}_{31}\varepsilon_x + \mathrm{C}_{32}\varepsilon_y + \mathrm{C}_{33}\varepsilon_z,\\
\tau_{yz} &= C_{44}\nu_{yz},\\
\tau_{zx} &= C_{55}\nu_{zx},\\
\tau_{xy} &= C_{66}\nu_{xy},
\end{aligned}\right\} \tag{2}$$

Hooke's law regarding the components of deformation will be

$$\begin{aligned}
\varepsilon_x &= a_{11}\sigma_x + a_{12}\sigma_y + a_{13}\sigma_z,\\
\varepsilon_y &= a_{21}\sigma_x + a_{22}\sigma_y + a_{23}\sigma_z,\\
\varepsilon_z &= a_{31}\sigma_x + a_{32}\sigma_y + a_{33}\sigma_z,\\
\nu_{yz} &= a_{44}\tau_{yz},\\
\nu_{zx} &= a_{55}\tau_{zx},\\
\nu_{xy} &= a_{66}\tau_{xy},
\end{aligned}$$

where C_{ik} are the elastic constants of the material, a_{ik} are the constants of elastic deformation,

$$a_{ik} = \mathrm{C}_{ik} / \Delta \ ,$$

where Δ is the determinant, composed of the coefficients of the right-hand side of Equation (2), C_{ik} are the corresponding minors of this determinant.

In the expanded form, we will have [1, 9]

$$a_{11} = \left(C_{22}C_{33} - C_{23}^2\right)/\Delta, \quad a_{22} = \left(C_{11}C_{33} - C_{13}^2\right)/\Delta, \quad a_{33} = \left(C_{11}C_{12} - C_{12}^2\right)/\Delta,$$

$$a_{12} = \left(C_{12}C_{33} - C_{13}C_{23}\right)/\Delta; \quad a_{13} = \left(C_{12}C_{23} - C_{13}C_{22}\right)/\Delta, \quad a_{23} = \left(C_{11}C_{23} - C_{13}C_{22}\right)/\Delta,$$

$$a_{44} = C_{44}^{-1}; \quad a_{55} = C_{55}^{-1}; \quad a_{66} = C_{66}^{-1},$$

where $\Delta = C_{11}C_{22}C_{33} - C_{12}C_{23}^2 - C_{12}^2C_{33} - C_{13}^2C_{22} + C_{12}C_{13}C_{23}$.

Physical and technical elastic constants are related by expressions

$$\left.\begin{array}{l}
E_x = a_{11}^{-1} = C_{11} - \left(C_{12}^2 C_{33} + C_{13}^2 C_{22} - 2C_{12}C_{13}C_{23}\right)\left(C_{22}C_{33} - C_{23}^2\right)^{-1};\\
E_y = a_{21}^{-1} = C_{22} - \left(C_{11}C_{23}^2 + C_{12}^2 C_{33} - 2C_{12}C_{13}C_{23}\right)\left(C_{11}C_{33} - C_{13}^2\right)^{-1};\\
E_z = a_{33}^{-1} = C_{33} - \left(C_{11}C_{23} + C_{13}^2 C_{22} - 2C_{12}C_{13}C_{23}\right)\left(C_{11}C_{22} - C_{12}^2\right)^{-1};\\
G_{yz} = a_{44}^{-1} = C_{44}; G_{xz} = a_{55}^{-1} = G_{55}; G_{xy} = a_{66}^{-1} = G_{66}^{-1};\\
\mu_{xy} = a_{12} / a_{22} = \left(C_{12}C_{33} - C_{13}C_{23}\right)\left(C_{11}C_{33} - C_{13}^2\right)^{-1};\\
\mu_{xz} = a_{13} / a_{33} = \left(C_{12}C_{23} - C_{13}C_{23}\right)\left(C_{11}C_{22} - C_{12}^2\right)^{-1};\\
\mu_{yx} = a_{12} / a_{11} = \left(C_{11}C_{33} - C_{13}C_{23}\right)\left(C_{22}C_{33} - C_{23}^2\right)^{-1};\\
\mu_{zx} = a_{23} / a_{22} = \left(C_{11}C_{23} - C_{13}C_{22}\right)\left(C_{11}C_{33} - C_{13}^2\right)^{-1};\\
\mu_{yz} = a_{23} / a_{33} = \left(C_{11}C_{23} - C_{13}C_{22}\right)\left(C_{11}C_{22} - C_{12}^2\right)^{-1};\\
\mu_{zy} = a_{23} / a_{22} = \left(C_{11}C_{23} - C_{13}C_{22}\right)\left(C_{11}C_{33} - C_{13}^2\right)^{-1},
\end{array}\right\} \quad (3)$$

where E_x, E_y, E_z, are elastic moduli; G_{yz}, G_{xz}, G_{xy} are shear moduli and $\mu_{xy}, \mu_{xz}, \mu_{yx}, \mu_{zx}, \mu_{yz}, \mu_{zy}$ are Poisson's ratios along the directions of elastic symmetry axes.

In the case of a plane stressed state, the elastic constants can be represented in the form [10, 12]

$$\begin{array}{l}
a_{11} = C_{22}\left(C_{11}C_{22} - C_{12}^2\right)^{-1},\\
a_{22} = C_{11}\left(C_{11}C_{22} - C_{12}^2\right)^{-1},\\
a_{66} = C_{66}^{-1}
\end{array}$$

Then the values of the elastic constants can be represented as:

$$\left.\begin{array}{l}
E_x = C_{11} - C_{12} / C_{22}; E_y = C_{22} - C_{12} / C_{11};\\
G_{xy} = a_{66}^{-1}; \mu_{yx} = C_{12} / C_{22};\\
\mu_{yx} = C_{12} / C_{11}
\end{array}\right\} \quad (4)$$

It should be noted that the elastic constants of the composite material can be determined by a non-destructive method by the parameters of the propagation of elastic waves [9, 11].

The above Equations (3) and (4) make it possible to calculate the elastic characteristics of the composite material only along the entire elastic symmetry. To determine the elastic characteristics of orthotropic materials in arbitrary directions, the following expressions are proposed [12, 13]

$$\begin{array}{l}
E_\varphi = E_x\left(\cos^4\varphi + b'\sin^2\varphi + \lambda\sin^4\varphi\right)^{-1},\\
\mu_\varphi = \left[\mu_{xy} - 0.25\left(1 + \lambda - 4b'\right)\sin^2 2\varphi\right]\left[\cos^4\varphi + b'\sin^2\varphi + \lambda\sin^4\varphi\right]^{-1},\\
G_\varphi = G_{xy}\left[1 - \left(1 - d\right)\sin^2 2\varphi\right]^{-1}, \lambda = E_x / E_y, b' = \dfrac{E_x}{E_{45}} - 0.25\left(\lambda + 1\right),\\
d = \left(1 + \lambda + 2\mu_{xy}\right)\left(4b' + 2\mu_{xy}\right)^{-1}.
\end{array}$$

By analogy with the anisotropic elastic characteristics in [4], expressions describing the anisotropic strength properties are proposed

$$\sigma_\varphi = \sigma_0\left(\cos^4\varphi + b\sin^2 2\varphi + C\sin^4\varphi\right)^{-1}, \quad (5)$$

where σ_φ is the ultimate tensile or compressive strength in an arbitrary direction; σ_0 is ultimate strength at pure shear at an 45^0 angle to the axis of elastic symmetry

$$c = \sigma_0 / \sigma_{90}; b = a - 0.25(нс); a = \sigma_0 / \sigma_{45} \ . \tag{6}$$

For ultimate strength at pure shear σ_0 in an arbitrary direction in the x,y plane an expression is suggested. On the basis of numerous experiments, good convergence of experimental and calculated data obtained by Equations (5) and (6) has been established. Of considerable interest are the conclusions about the strength made on the basis of the generalized Goldenblatt - Kapnov criterion, which has the below form [6-10]

$$1/\sigma_\varphi^p = \Pi_{11}^0 \cos^2\varphi + \Pi_{22}^0 \sin^2\varphi + \sqrt{\Pi_{11}^0 \cos^4\varphi + \Pi_{2222}^0 \sin^4\varphi + \left(\Pi_{1212}^0 + 0.5\Pi_{1122}^0\right)\sin^2\varphi};$$

$$1/\sigma_\varphi^c = -\Pi_{11}^0 \cos^2\varphi - \Pi_{22}^0 \sin^2\varphi + \gamma\Pi_{11}^0 \cos^4\varphi + \Pi_{2222}^0 \sin^4\varphi + \left(\Pi_{1212}^0 + 0.5\Pi_{1112}^0\right)\sin^2\varphi;$$

$$1/\sigma^+ = \left(\Pi_{11}^0 + \Pi_{22}^0\right)\sin^2\varphi + \sqrt{(\Pi_{1111}^0 - \Pi_{2222}^0 - 2\Pi_{1122}^0)\sin^2 2\varphi + 4\Pi_{1212}^0 \cos 2\varphi};$$

$$1/\sigma^- = -\left(\Pi_{11}^0 - \Pi_{22}^0\right)\sin^2\varphi + \sqrt{(\Pi_{1111}^0 - \Pi_{2222}^0 - \Pi_{1122}^0)\sin^2 2\varphi + 4\Pi_{1212}^0 \cos^2 4\varphi};$$

$$\Pi_{11}^0 = 0.5\left(1/\sigma_\varphi^p - 1/\sigma_0^c\right); \Pi_{22}^0 = 0.5\left(1/\sigma_{90}^p - 1/\sigma_{90}^c\right); \Pi_{1111}^0 = 0.25\left(1/\sigma_0^p - 1/\sigma_0^c\right);$$

$$\Pi_{2222}^0 = 0.25\left(1/\sigma_{90}^p - 1/\sigma_{90}^c\right); \Pi_{1122}^0 = 0.125\left(1/\sigma_0^p - 1/\sigma_0^c\right)^2 + \left(1/\sigma_{90}^p - 1/\sigma_{90}^c\right)^2 - \left(1/\tau_{45}^+ + 1/\tau_{45}^-\right)^2;$$

$$\Pi_{1212}^0 = 0.0625\left(1/\tau_0^+ + 1/\tau_0^-\right)^2.$$

Strength anisotropy according to the Goldenblatt - Kapnov criterion can be used only after strength indicators are experimentally determined.

The dependence of the strength of composite materials on the direction of testing according to [2-5] has the form

$$\sigma_\varphi = \sigma_0\left(\sqrt{\cos^2\varphi + a^{-2}\sin^2\varphi + 0.5\left(2b^{-2} - a^{-2} - 1\right)\sin^2 2\varphi}\right)^{-1},$$

where $a = \sigma_{90} / \sigma_0; b = \sigma_{45} / \sigma_0$.

Conclusion

In industry, structures are widely used, where strength determines their performance. Among such structures are hollow bodies of revolution (pipes, tanks, cylinders, etc.). Stress states arise in these structures under the action of operational loads. The peculiarity of the composite material role in the structure is to minimize the stress level perpendicular to the plane of the reinforcing layers. Under uniaxial stress of composite material specimens, the axes of which lie in the plane of reinforcement and make an angle α with the axes of elastic symmetry of the material (in relation to the turns directions) the stresses will be equal (Fig.)

$$\sigma_x = \sigma_b \cos^2\alpha; \sigma_y = \sigma_b \sin^2\alpha; \sigma_{xy} = 0.5\sigma_b \sin^2 2\alpha \ .$$

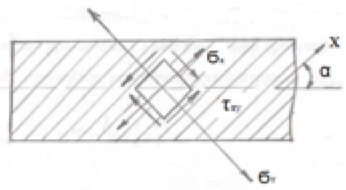

Fig. *Scheme of the stress state in the sample under tension at an angle to the axes of elastic symmetry of the composite material*

If the degree of a composite anisotropy $(\sigma_{90}/\sigma_0; \sigma_{45}/\sigma_0)$ is known, then it is enough, for a given material, to experimentally determine only one strength characteristic, for example, σ0.

At that the ultimate strength in any direction is determined by the Equation (5).

For non-destructive strength tests of composite products, the optimal strength criterion will be that which can be expressed through the anisotropy index determined directly in the product in different structural directions without destroying them.

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