Victor Dmitriy Eryomin

Don State Technical University (DSTU), Rostov-on-Don, RF

ON NATURAL OSCILLATIONS OF A THIN ELASTIC WAVY SHELL OF AN OPEN PROFILE

The article touches upon the problem of natural oscillations of a thin elastic wavy shell of an open profile. The proposed method for determining the numerical values of the lowest frequencies and the corresponding forms of natural oscillations of shells of a complicated shape is based on the Rayleigh-Ritz energy method.

The results of numerical calculation of this thin wavy shell with a hinge-fixed attachment along the lower contour along the generatrix are presented.

Keywords: *wavy thin elastic shell, the frequency and form of natural oscillations, the energy method.*

Introduction

The experience of construction in our country and abroad increasingly points to prospects of using shells with a complicated form, including wavy shells, as coatings of public, industrial, warehouse and agricultural buildings and structures.

The creation of new, more advanced engineering structures led to the necessity to develop the theory of calculation of shells with a complex shape: layered and corrugated, wavy and supported by a rod set.

The theory of thin shell calculation is well developed, so it is possible to calculate and then design the installations and structures in the form of shells of rather complex outlines.

However, in many cases they are exposed to periodic and impulsive loads, especially in seismic areas, so it is practically very important to have a simple mathematical apparatus to determine own frequencies of shells with complex geometry and natural oscillation forms corresponding to them.

By analyzing scientific works on the dynamics of shells, it is easy to notice the presence of a relatively small number of studies on calculation of natural oscillations of shells of nonclassic form.

The method of determination of numerical values of the lowest frequencies and natural oscillations of thin shells of complex shape corresponding to them by the energy method is proposed, which is acceptable for use in design practice.

This method allows replacing the differential equations with a uniform system of linear algebraic equations, which greatly simplifies the dynamic calculation of a shell of complex structure. As a particular case, this technique is applied for calculating smooth shells with any boundary conditions.

Task Setting

The problem of determination of numerical values of the lowest frequencies and natural oscillations of thin elastic wavy shell rectangular in plan with hinge-fixed attachment along the lower contour along the generatrix is considered (Fig.1).

Cross - section of the medial surface of the shell (by crest) is outlined by a curve, the equation of which is:

$$\frac{x^2}{a^2} + \frac{y^3}{b^3} = 1$$

$$\text{(1)}$$

Medial surface of the shell is formed by curve movement

$$\alpha = -\Delta \left(1 - \cos \pi \frac{z}{\ell}\right)$$

$$\text{(2)}$$

along two adjacent crests of the shell and located in the normal flatness towards them. Here Δ is the cosine wave amplitude.
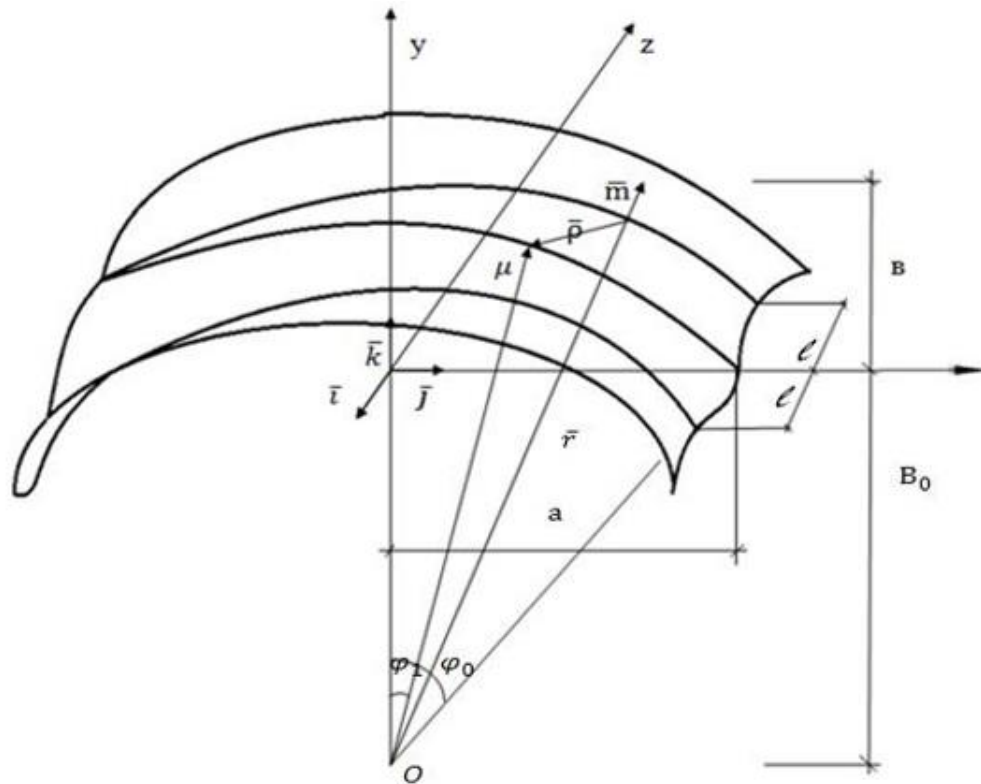
Fig. 1.

Geometric Characteristics (shell sizes)

Span - 2a = 18

Height in cross-section passing through the crest of the shell – h = 5*m*.

Wavelength - 2ℓ = 3*m*.

Wave amplitude - Δ = 0.225*m*.

Pole distance - B$_o$ = 9*m*.

Shell thickness - 2h = 0.05*m*.

Physical Characteristics:

Shell material elasticity module - E = 28*HPa*

Poisson's ratio - $\mu = \dfrac{1}{6}$.

Shell material density - ρ = 2500*kg/m³*.

Goal of the Paper

Obtaining numerical values of the lowest frequencies and natural oscillations of thin elastic wavy shell rectangular in plan corresponding to them (Fig. 1), with hinge-fixed attachment along the lower contour along the generatrix with coordinate $\alpha_2 = \pm 1$ (Fig.2).

The energy method of Ritz-Relay is the basis of the proposed methodology for determining the lowest frequencies and natural oscillation forms of complex shells corresponding to them. This method allows to obtain sufficiently precise values of the lowest frequencies and natural oscillations of complex shells corresponding to them under arbitrary conditions of fixation and under any law of changes in its geometric and physical characteristics. The main provisions of this methodology are outlined in [1-12].
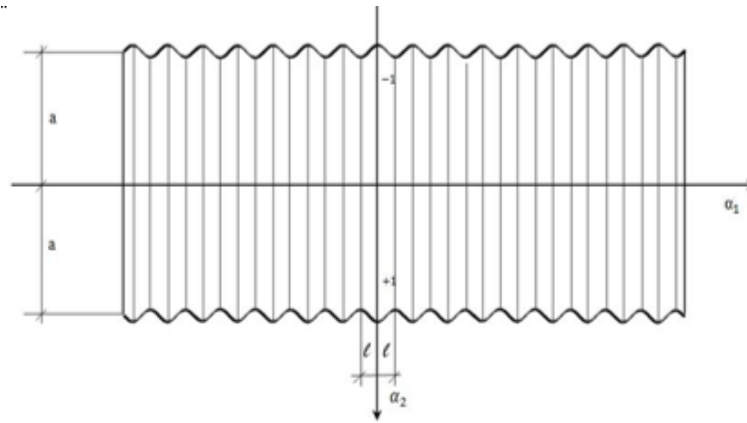
Fig. 2.

Calculation is based on geometric and physical linearity using Kirchhoff-Love hypothesis. It is assumed that the length of the shell along the Z axis (Fig.1) is rather large, so it is possible to limit the examination to the middle parts of it, without taking into account the influence of the shell parts adjacent to its ends.

Vector equation of the median surface of this wavy shell is obtained. Formulas for calculating parameters of Lamé and Christoffel symbols of this shell are displayed [6].

Functions approximating amplitudes of movement of points of the median surface of the shell along curvilinear axes of coordinates, in the form of double trigonometric rows, satisfying the conditions of hinge-fixed attachment of the shell along the lower contour along the generatrix are selected, as $\alpha_2 = \pm1$: $u_1^0 = u_2^0 = u_3^0 = 0$ (Fig.2).

According to the formulas given in [7], matrices necessary for determination of numerical values of lower frequencies and natural oscillations of the given wavy shell are obtained.

Numeric Example Solution

The control of numerical convergence of the proposed algorithm calculation of the wavy shell is shown in Fig. 1, was carried out by the method of successive approximations, by stepwise increasing the number of members of the row approximating the solution.

In the first approximation in double trigonometric rows approximating the movement, four members have been left (*m=n=2*), in the second approximation – nine (*m=n=3*), then sixteen (*m=n=4*), twenty five (*m=n=5*) and finally thirty six members in the row (*m=n=6*). Comparing the last three approximations in *m=n=4;5;6,* we will get convergence of results sufficient for engineering calculations.

The results of calculating the first two frequencies and forms of natural oscillations of the wavy shell with hinge-fixed attachment along the lower contour corresponding to them are shown in graphs 1-14.
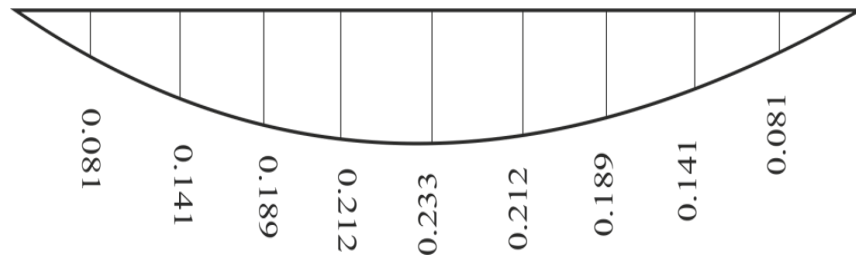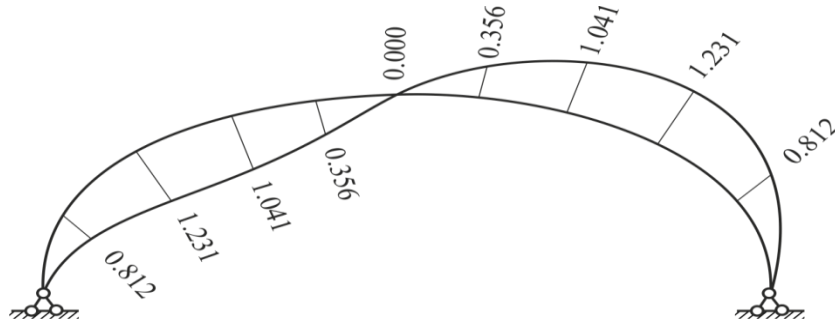
First Frequency

$\omega_1 = 49.2 Hz$

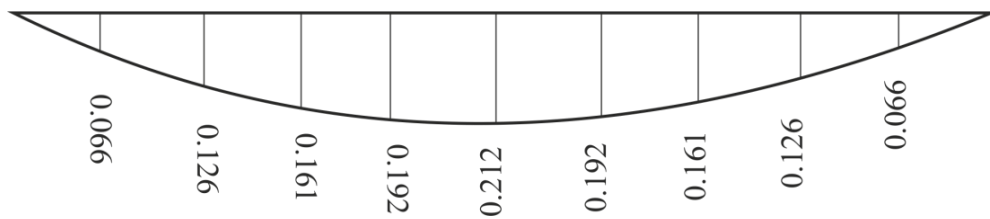**First Form of Natural Oscillations**
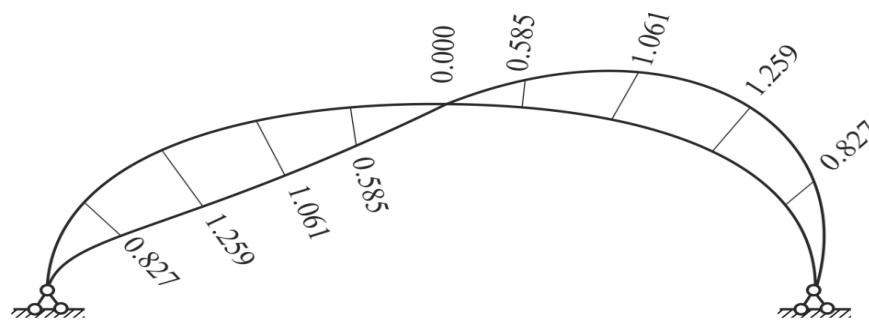
Graph 1. *Amplitude values of non-dimensional movements*
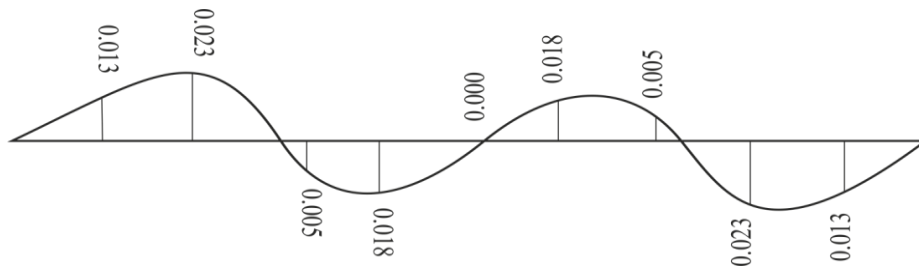u_3^0 *in cross sections of a shell with coordinates* $\alpha_1 = 0$

20

Graph 2. *Amplitude values of non-dimensional movements u_2 in cross – sections of the shell with coordinates $\alpha_1 = 0$*
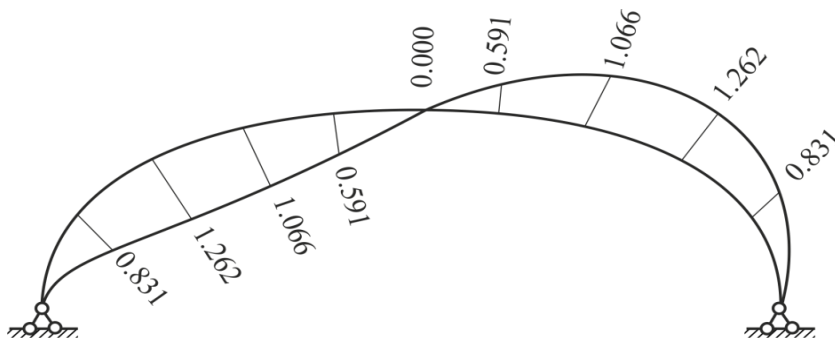

Graph 3. *Amplitude values of non-dimensional movements u_3^0 in cross- sections of a shell with coordinates $\alpha_1 = \pm 0.5$*
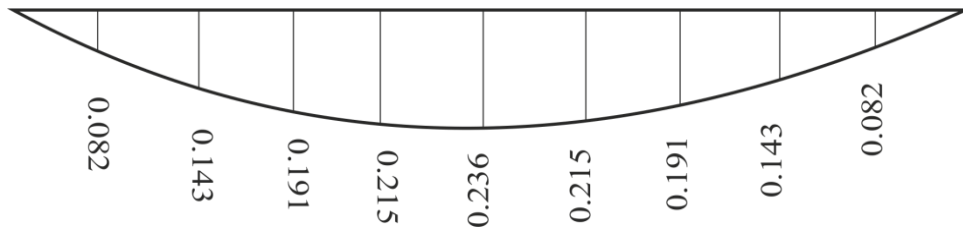

Graph 4. *Amplitude values of non-dimensional movements u_2^0 in cross -sections of a shell with coordinates $\alpha_1 = \pm 0.5$*

Graph 5. *Amplitude values of non-dimensional movements u_1^0 in cross- sections of a shell with coordinates $\alpha_1 = \pm 0.5$*

Graph 6. *Amplitude values of non-dimensional movements u_1^0 in cross -sections of a shell with coordinates $\alpha_1 = \pm 0.5$*
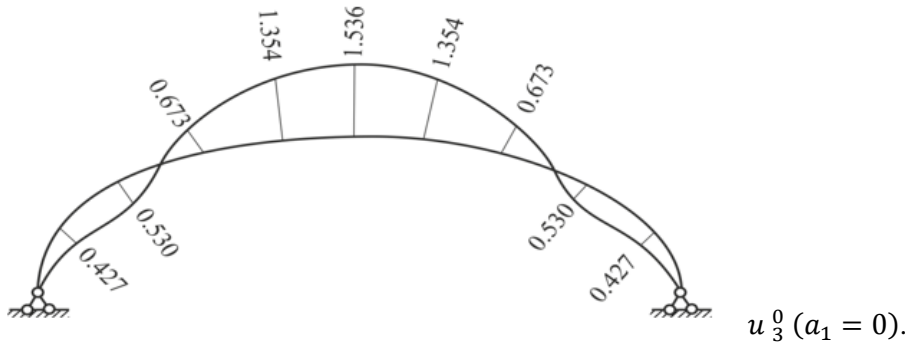
Graph 7. *Amplitude values of non-dimensional movements*
u_2 *in cross- sections of a shell with coordinates* $\alpha_1 = \pm 1$

Amplitude values of non-dimensional movements u_1^0 in sections of a shell with coordinates $\alpha_1 = 0$ and $\alpha_1 = \pm 1$ are equal to zero.
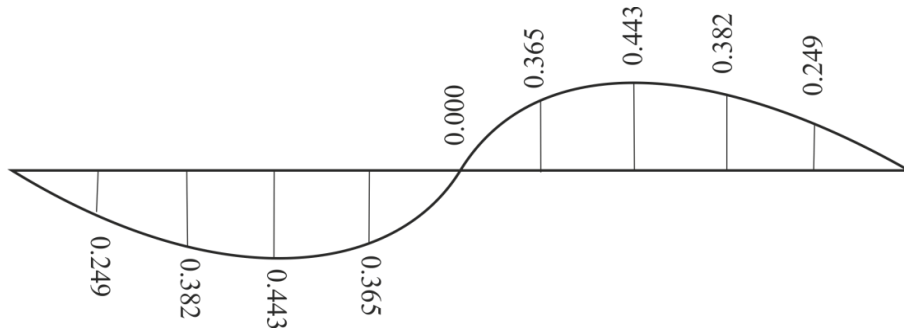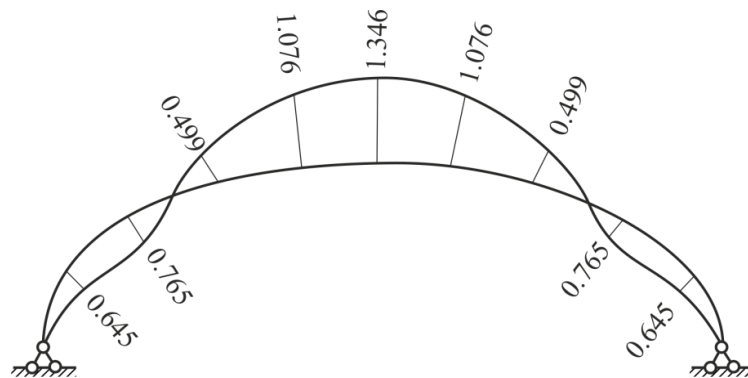
Second part

$\omega_2 = 58.4$ *Hz.*

Second Form of Natural Oscillations

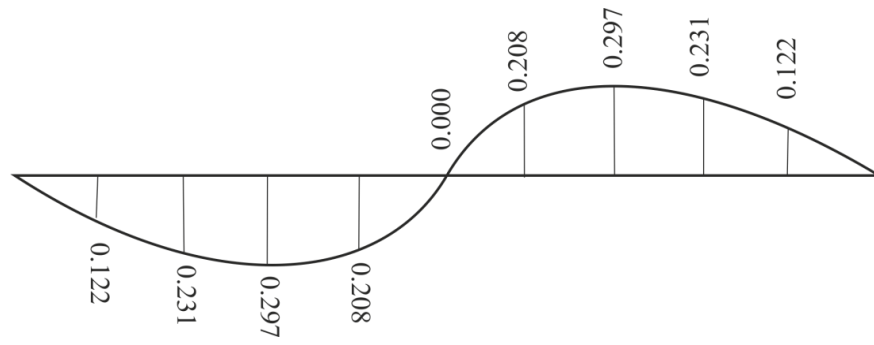$u_3^0 (a_1 = 0).$

Graph 8. *Amplitude values of non-dimensional movements*
u_3^0 *in cross- sections of a shell with coordinates* $\alpha_1 = 0.$

Graph 9. *Amplitude values of non-dimensional movements* u_2
in cross- sections of a shell with coordinates $\alpha_1 = 0$

Graph 10. *Amplitude values of non-dimensional movements* u_3^0
in cross- sections of a shell with coordinates $\alpha_1 = \pm 0.5$

22

Graph 11. *Amplitude values of non-dimensional movements* u_2^0
in cross- sections of a shell with coordinates $\alpha_1 = \pm 0.5$
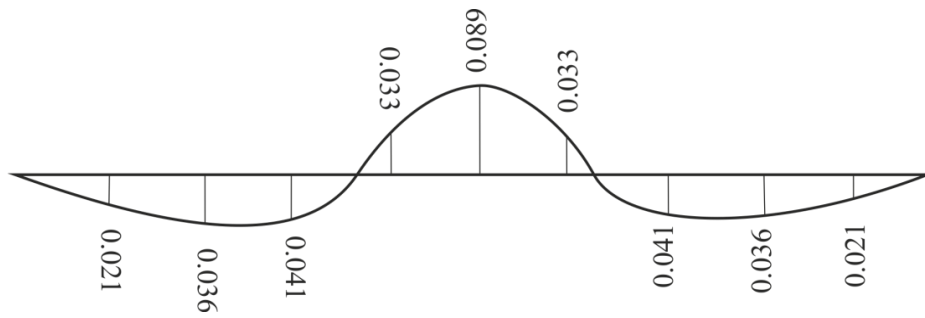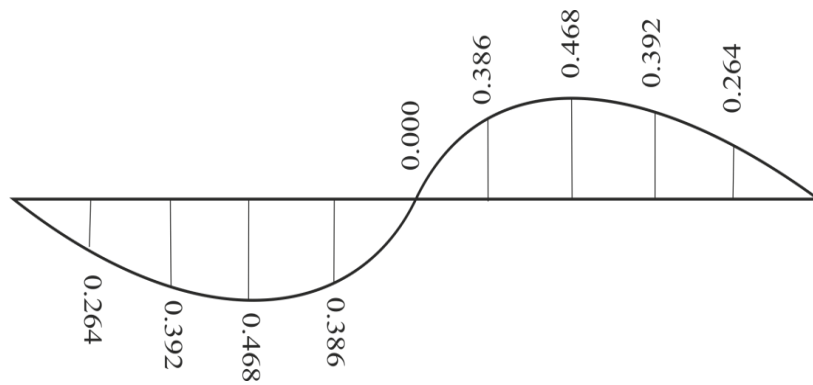

Graph 12. *Amplitude values of non-dimensional movements* u_1^0
in cross- sections of a shell with coordinates $\alpha_1 = 0.5$
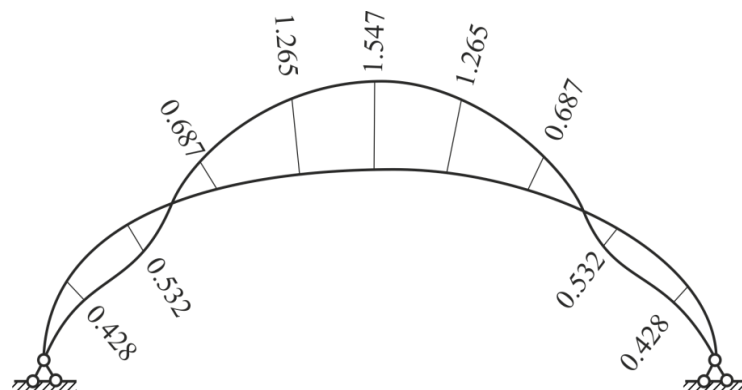

Graph 13. *Amplitude values of non-dimensional movements* u_2
in cross - sections of a shell with coordinates $\alpha_1 = \pm 1$

Graph 14. *Amplitude values of non-dimensional movements* u_3^0
in cross - sections of a shell with coordinates $\alpha_1 = \pm 1$

Amplitude values of non-dimensional movements u_1^0 in cross-section of the shell with coordinates $\alpha_1 = 0$ and $\alpha_1 = \pm 1$ are equal to zero.

On this graphs ω_1, ω_2 are the frequencies of natural oscillations of thin wavy shells, u_1^0, u_2^0, u_3^0 are the amplitudes of non-dimensional movements of the points of shell median surface along the axes of coordinate α_1, α_2 and α_3 accordingly.

Conclusion

Analyzing the obtained values of the first three frequencies of natural oscillations of the thin wavy shell with hinge-fixed attachment of the shell along the lower contour along the generatrix, it can be noted that in this case also the fifth approach (36 members of the row) with sufficient engineering accuracy allows determining the values of the lowest frequencies. The difference between the fourth and fifth approximations by the first frequency is 2.2%, by the second is 3.4%, by the third is 4.0%. The obtained values of the lowest frequencies of the natural oscillations of the given thin wavy shell confirm good convergence of the algorithm in this case as well.

Comparison of the obtained results of calculation of these wavy shells for two options of its fixation with the rigidly caught lower contour [11] and hinge-fixed attachments shows that in the case of hinge-fixed attachment of the shell its lowest frequencies of natural oscillations have decreased by 10-16%.

In both cases, two transverse half-waves correspond to the main tone of the natural oscillations thin wavy shell.

The results of numerical calculation of this shell allow recommending the application of the proposed methodology for determination of the lowest frequencies and forms of natural oscillations of shells of complex form corresponding to them.

References

[1]. L.V. Kantorovich, V.I. Krylov, Priblijionnye metody vysshego analisa. Fizmatgiz, Moscow, 1962 (in Russian).
[2]. A.L. Goldenveiser, Teoriya uprugikh tonkikh obolochek. Nauka, Moskow, 1976 (in Russian).
[3]. K.B. Aksentjan, V.D. Eryomin, Printsip vosmojnykh peremesceniy v sluchae svobodnykh kolebaniy. Raschjot obolochek i plastin, 1977 (in Russian).
[4]. K.B. Aksentjan, V.K. Gordeev – Gavrikov, Variatsionno – energeticheskiy metod raschjota kolebaniy injenernykh soorujeniy. RGU, Rostov – on – Don, 1979 (in Russian).
[5]. V.D. Eryomin, Opredelenie chastot i form sobstvennykh kolebaniy obolochek neklassicheskoy formy. Scientific papers of National University of Architecture and Construction of Armenia, 1, 2015, 94-100 (in Russian).
[6]. V.D. Eryomin, Sobstvennye kolebaniya nekrugovoy cilindricheskoy uprugoy volnistoy obolochki otkrytogo profilia. Scientific papers of National University of Architecture and Construction of Armenia, 1, 2015, 101-108 (in Russian).
[7]. V.D. Eryomin, K raschyotu sobstvennykh kolebaniy tonkoy volnistoy obolochki otkrytogo profilya. Scientific papers of National University of Architecture and Construction of Armenia, 1, 2016, 64-71 (in Russian).
[8]. V.D. Eryomin, Issledovanie chislennoy skhodimosti algoritma opredeleniya nisshikh chastot i form sobstvennykh kolebaniy obolochek uslojnyonnoy formy. Scientific papers of National University of Architecture and Construction of Armenia, 1, 2016, 72-77 (in Russian).
[9]. V.D. Eryomin, K voprosu ob issledovanii chislennoy skhodimosti algoritma opredeleniya nisshikh chastot i form sobstvennykh kolebaniy obolochek uslojnyonnoy formy. Scientific papers of National University of Architecture and Construction of Armenia, 3, 2017, 50-55 (in Russian).
[10]. V. D. Eryomin, The Study of Natural Oscillations of Thin Elastic Shells of Complicated Shapes by the Energy Method. Materials Science Forum, 931, 2018, 42-46.

[11]. V.D. Eryomin, Opredelenie chastot i form sobstvennykh kolebaniy tonkoy uprugoy volnistoy obolochki otkrytogo profilia. Scientific papers of National University of Architecture and Construction of Armenia, 1, 2019, 26-34 (in Russian).

[12]. V.D. Eryomin, K voprosu opredeleniya nisshikh chastot i form sobstvennykh kolebaniy tonkoy volnistoy obolochki otkrytogo profilia. Scientific papers of National University of Architecture and Construction of Armenia, 4, 2019, 20-26 (in Russian).

Victor Dmitriy Eryomin, doctor of philosophy (PhD) in engineering, professor (RF, Rostov-on-Don) - Don State Technical University (DSTU), professor at the Chair of Strength of Materials, eremin.vd@yandex.ru