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FORMATION OF STIFFNESS MATRICES OF PLANE ELEMENTS BASED ON HIGH-ORDER DEGREE POLYNOMIALS

In this paper, the method for forming the stiffness matrix of a plane element in a plane stress-strain state using functions approximating the displacements of the points of the element in the form of high-order degree polynomials is considered.

Keywords: *stiffness matrix, Ritz method, stress-strain state, potential strain energy.*

Introduction

When calculating structures using the finite element method, a solid area is represented as an area consisting of a number of smaller areas, called finite elements. To approximate the point displacements of the finite rectangular or triangular element, linear functions are usually used to ensure the continuity of deformations in the nodes and along the edges of the junction of finite elements. In order to increase the accuracy of the solution, the number of finite elements into which a solid area is divided increases, resulting in a significant increase in the order of the system of solving equations.

When calculating a lamellar structure, for example a large-panel building, the main structural elements of which are plane rectangular elements (wall panels, partition panels, floor and roof slabs), a typical structural element of a large-panel building can be taken as a finite element.

Since during the assembly of a large-panel building the main docking of its elements is carried out at nodal points, when selecting functions approximating the displacement of points of the element, it is not necessary to ensure conditions of continuity of deformations along the edges of structure element dockings. This allows using nonlinear functions as approximating functions, for example, high-order degree polynomials. The basic provisions of this methodology are described in [1-3].

Stiffness Matrix of a Plane Element Based on High-Order Polynomials

In the work [4], the stiffness matrix of a plane rectangular plate was obtained, operating according to the beam-wall scheme, loaded with concentrated forces applied at the nodal points of the plate free from bonds. The problem was solved by the Ritz variation-energy method. To approximate the displacement functions of an arbitrary point, degree polynomials of the following type were taken:

$$u_1 = f_1(x,y) \sum_{m=0}^{K}\sum_{n=0}^{K} \alpha_{mn}^{I} x^m y^n;$$

$$u_2 = f_2(x,y) \sum_{m=0}^{K}\sum_{n=0}^{K} \alpha_{mn}^{II} x^m y^n, \qquad (1)$$

where

$K = (m+1)\cdot(n+1)$ – number of members of the series to be retained;

$f_1(x,y)$ and $f_2(x,y)$ – functions that meet kinematical boundary conditions (conditions for fixing the plate as a completely solid body).

The accuracy of the solution was achieved by increasing the number of retained members of series. In this case, the dimension of the obtained stiffness matrix remained constant.

In the finite element method, the elements into which the structure is divided are outwardly kinematically free. In order to use the considered plate as a finite element of the system, we will free it from bonds and allow it to move in its plane, as an absolutely stiff body (Fig. 1).

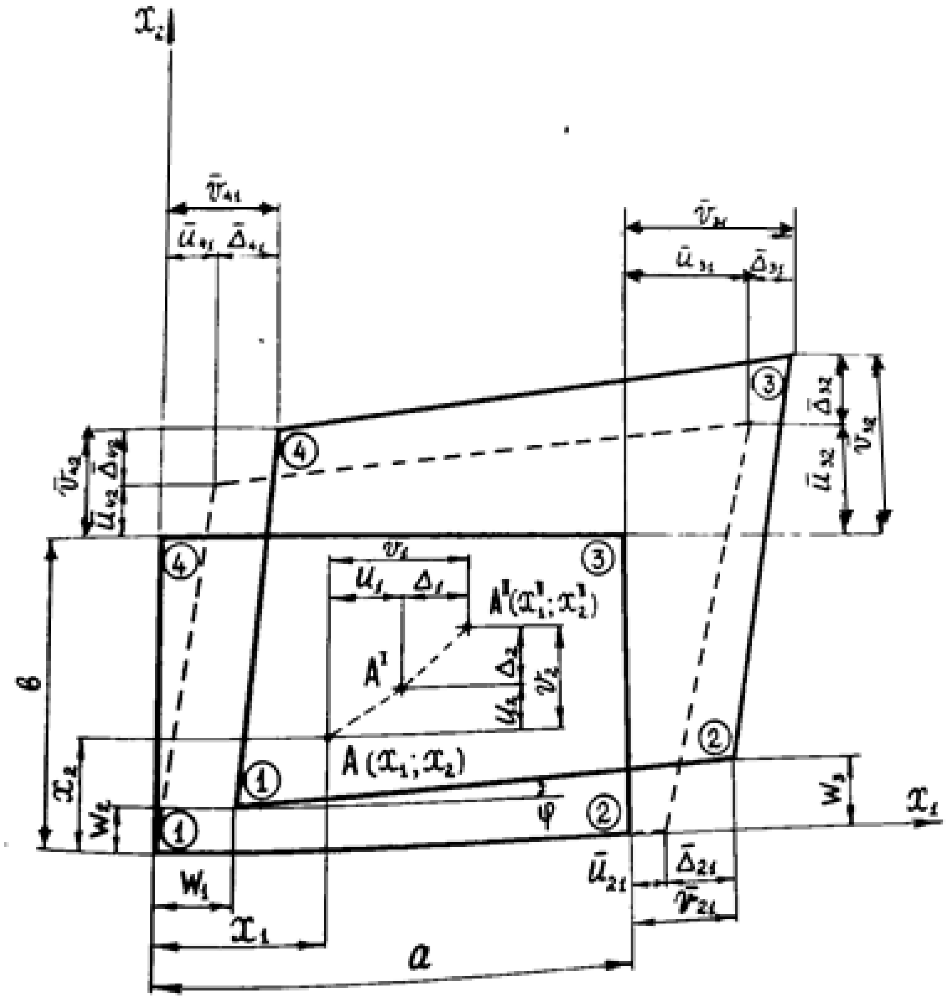

Fig 1.

Here

u - displacements of the arbitrary point caused by elastic deformations;

Δ - additional displacements of the arbitrary point caused by the displacement of an element as an absolutely solid body;

v - full displacements of the arbitrary point;

w_1, w_2, w_3 - components of the "rigid" displacement of the plate.

During small displacements

$$tg\varphi = \varphi = (w_3 - w_2) / a.$$

The total displacements of the arbitrary point A (x1, x2) inside the plate will consist of displacements caused by elastic deformation and additional displacements caused by the displacement of the plate as an absolutely solid body:

$$\underset{(2\times1)}{\{v\}} = \underset{(2\times1)}{\{u\}} + \underset{(2\times1)}{\{\Delta\}}. \qquad (2)$$

The vector of elastic displacements {u} was obtained in [4].

Additional displacements Δ are expressed through w displacements according to the known ratios of analytical geometry (Fig. 2).

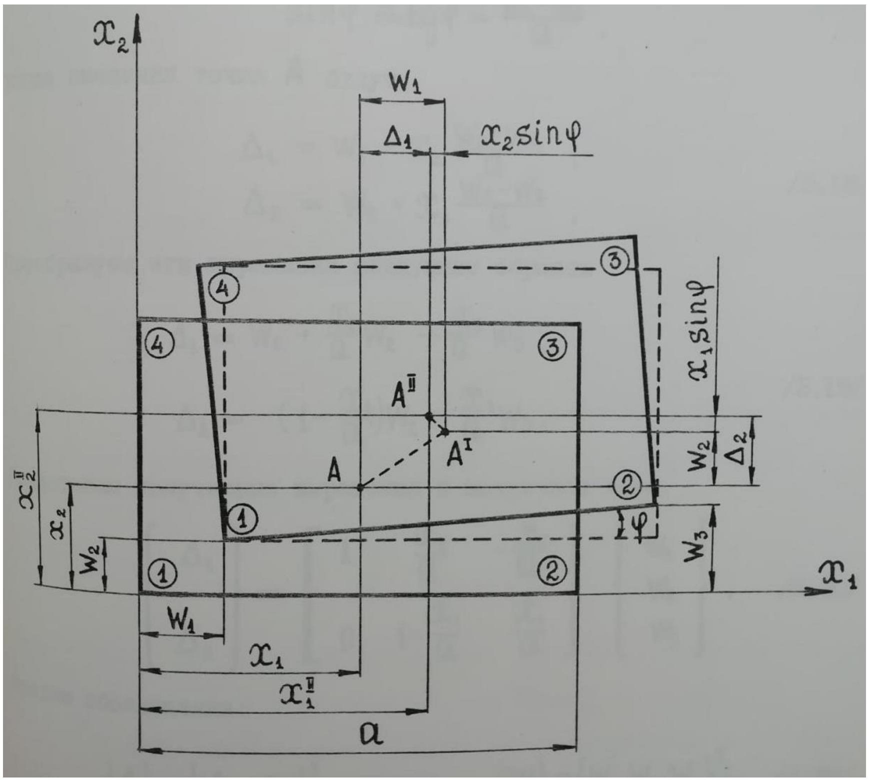

Fig. 2.

The displacement of the considered point A after displacement of the plate as an absolutely solid body will be equal to:

$$\Delta_1 = x_1^{II} - x_1 = w_1 + x_1 cos\varphi - x_2 sin\varphi - x_1 = w_1 - x_2 sin\varphi - x_1 2sin^2\varphi/2;$$

$$\Delta_2 = x_2^{II} - x_2 = w_2 + x_1 sin\varphi + x_2 cos\varphi - x_2 = w_2 - x_1 sin\varphi - x_2 2sin^2\varphi/2. \quad (3)$$

Taking into account the smallness of the angle φ, it is possible to neglect the sizes of the second order of smallness in expression (2) and, besides, to accept

$$\sin\varphi \approx tg\varphi = (w_3 - w_2) / 2.$$

Then the expressions for the displacement of point A can be recorded as follows:

$$\Delta_1 = w_1 + x_2 \cdot w_2 / a - x_2 \cdot w_3 / a;$$

$$\Delta_2 = (1 - x_1 / a)\, w_2 + x_1 \cdot w_3 / a. \quad (4)$$

Let us represent the resulting equations in matrix form:

$$\{\Delta\} = [Z_D] \cdot \{w\}. \quad (5)$$

Here the matrix $[Z_D]$ of dimension (2×3) has the following form:

$$[Z_D] = \begin{Vmatrix} 1 & x_2/a & -x_2/a \\ 0 & 1 - x_1/a & x_1/a \end{Vmatrix}$$

Put the obtained expressions for {u} and $\{\Delta\}$ in formula (1):

$$v\} = [Z] \cdot \{U\} + [Z_V] \cdot \{w\}. \quad (6)$$

$$(2\times1)\ \ (2\times3)\ (5\times1)\ (2\times3)\ (3\times1)$$

After a simple transformation, expression (5) is represented as:

$$\{v\} = [Z_d]\cdot\{p\}, \qquad (7)$$

$$(2\times1)\ (2\times8)(8\times1)$$

where {p} - a vector, the elements of which are "elastic" displacements of unfixed nodes of the plate and three independent displacements of this plate as an absolutely solid body.

$$\{p\} = \{u_{21}\ u_{31}\ u_{32}\ u_{41}\ u_{42}\ w_1\ w_2\ w_3\}^T. \qquad (8)$$

By setting the coordinates of the nodal points in formula (4), the total nodal displacements are expressed through the vector {p}:

$$\{V_0\} = [M]\cdot\{p\}. \qquad (9)$$

$$(8\times1)\ \ (8\times8)(8\times1)$$

Matrix [M] is obtained from consideration of the geometric scheme of displacements. The vector {p} is determined from expression (8):

$$\{p\} = [\ M\ ]^{-1}\{\ V_0\ \}. \qquad (10)$$

By putting the expression (9) into formula (6), we will obtain the value of the displacements of the arbitrary point of the plate through the displacements of its nodes:

$$\{v\} = [\ Z_п\ ]\ [\ M\ ]^{-1}\{\ V_0\ \} \quad \text{or} \quad \{v\} = [\ Z_0\ ]\{\ V_0\ \}, \qquad (11)$$

$$(8\times1)\ \ (8\times8)\ (8\times1)$$

where $[\ Z_0\ ] = [\ Z_п\ ]\ [\ M\ ]^{-1}$ is the so-called coordinate matrix.

Expressing all the components of the stress-strain state of the arbitrary point of the plate through the displacements of its nodes, we make up the functional of the total potential energy of the deformation of a rectangular flat plate having 8 degrees of freedom and loaded with a system of nodal concentrated forces.

Minimizing the functional of the potential deformation energy through nodal displacements, we come to a system of algebraic equilibrium equations:

$$\frac{Eh}{1-\nu^2}[\ T\ ]\cdot\{U\} = \{\ F\ \}. \qquad (12)$$

$$(8\times8)\ (8\times1)\ (8\times1)$$

Here

E and ν – elastic constants of the material;

h –plate thickness;

{ F } – vector of nodal concentrated forces.

Rectangular symmetric matrix [T] is the stiffness matrix of the standard element:

$$[\ T\ ] = \iint [S]^T\ [\ \mu]\cdot[\ S\ ]dx_1dx_2. \qquad (13)$$

Here [S] and [μ] - matrices obtained by forming an expression for the potential energy of deformations in matrix form.

The general algorithm for calculating the elements of the stiffness matrix is represented by the formula:

$$t_{ij} = \iint [(s_{1i} + \mu s_{2i})s_{1j} + (\mu s_{1i} + s_{2i})s_{2j} + \frac{1-\mu}{2} s_{3i}s_{3j}]dx_1dx_2. \qquad (14)$$

Conclusion

The stiffness matrix of the element obtained by the given algorithm depends on the degree of approximation of the displacement functions. The desire to obtain a more accurate solution will require the preservation of a larger number of members of the series (1).

In traditional FEM, the refinement of the solution is achieved by dividing the solid area into smaller parts or by assigning a greater number of node points to the finite element. But if in the FEM this leads to an increase in the size of the stiffness matrices, then they have a constant dimension regardless of the number of retained members of the series in the displacement functions.

This allows taking relatively large structural elements of large-panel buildings and other prefabricated structures as basic finite elements, which makes it possible to significantly reduce the order of the resolving system of equations formulated for the entire structure.

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