PREDICTION OF SYNTHETIC SEISMOGRAMS AND ACCELEROGRAMS FOR TWO-LAYER BASIS BEDDINGS

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Abstract: *A method has been developed for predicting strong ground motion displacements and accelerations, assuming that an earthquake is an instantaneous mechanical rupture of the Earth's crust. The method uses derived theoretical formulas to calculate all three parameters of the ground motion: displacements, velocities, and accelerations during strong (with a magnitude of M≥6.0) earthquakes for any non-homogeneous (multilayer) ground beddings with various physical and mechanical characteristics – thicknesses, densities, and shear moduli – and at a certain distance from the expected earthquake's rupture. The example provided involves the results obtained for a number of two-layer heterogeneous site variants in seismic categories I-IV at the magnitude of M=7.0 and distance of 15 km from the expected earthquake's rupture. A comparison of the results obtained for actual heterogeneous foundation beddings with the equivalent homogeneous beddings showed divergences by 1.3-1.6 times, depending on the number of higher mode oscillations considered. Recommendations are provided for simplified calculation of seismograms and accelerograms for heterogeneous foundation beddings, with a certain correction of calculation results for equivalent homogeneous beddings.*

Keywords: *Prediction of synthetic seismograms and accelerograms, heterogeneous foundation (basis) bedding, seismic categories of soils.*

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Introduction

One of the main issues in seismic-resistant construction is the availability of a large number of accelerograms for past strong earthquakes at the site of a future construction project, recorded during a sufficiently long period. This would allow to identify statistically characteristic types of accelerograms for the given site and, therefore, to design reliable structures through calculations of seismic impact based on the selected accelerogram or a group of accelerograms. However, so far there have been too little records of past strong earthquakes in the seismically active zones of the Earth [1,2].

During an earthquake, in addition to the permanent (dead load weight) and imposed (weight of what is inside the building, rooftop snow, etc.) loads, buildings and structures are subjected to seismic loads. In reality, the latter are not loads (forces) per se. When the ground moves under a structure, due to inertia its individual parts move with a lag compared to the ground motion and the structure bends. This is similar to the impact of horizontal forces on the structure, which are perpendicular to its axis. These inertia forces are caused by the mass and rigidity of the superstructure. Values of these alternating inertia or seismic forces depend primarily on the alternating values of ground acceleration (accelerogram) during the earthquake. All of the above mentioned loads, except the seismic ones, create direct physical impact, have permanent direction, and exist during the whole useful lifespan of a structure. Seismic load has a dynamic nature and is active only during an earthquake. Therefore, seismic hazard parameters for a given area primarily depend on the values of horizontal (vertical, rotational) ground motion accelerations and their changes over the time [3]. This proves the importance of studying the issues related to creation of artificial seismograms and accelerograms for strong earthquakes that would adequately reflect the properties of actual recorded earthquakes. A brief overview of the problem was published in the initial articles [3,4]. The results of the studies published in the articles (in the opinion of many experts, close to the results recorded during real strong earthquakes) are apparently the only ones at the moment.

The choice of a two-layer base as an example of a heterogeneous base is due to the fact that for two-layer base the ratio V_{s1} and V_{s2} of the shear wave propagation velocities in soils during strong earthquakes plays a significant role in the magnitude of displacements and accelerations of the Earth's soil surface, and in many cases the cause of destruction of above-ground structures is the low value of the first layer velocity V_{s1}, (in some cases it is advisable to remove the "weak" first layer). In addition, any conventional multi-layer base can be reduced to a two-layer base with equivalent dynamic and seismic properties. Therefore, the main goal of this study is to cover all possible cases of different ratios V_{s1} and V_{s2} of two-layer bases (the article considers 16 cases). It should also be noted that in developing the forecasting method [3,4] the fundamental works of the following famous scientists were widely used: Brune J., Esteva L., Kasahara K., Wells D. and Coppersmith K., Lomnitz C. and Singh K., Faccioli E. and Resendiz D., etc [5-9].

Materials and Methods

In our previous works [3,4] a method was developed to predict strong ground motion displacements and accelerations, assuming that an earthquake is an instantaneous mechanical rupture of the Earth's crust (Fig.1.).

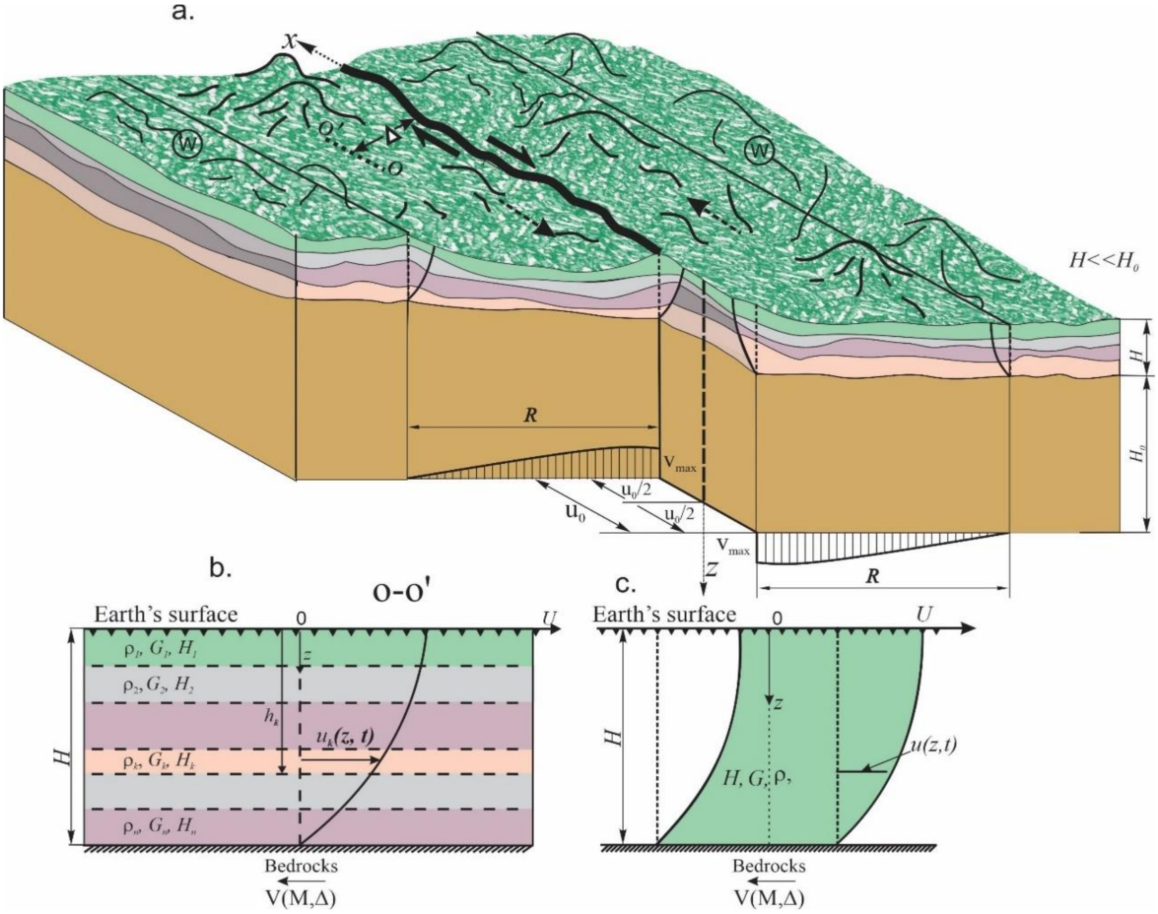


Fig. 1. *Schematic diagram of the medium after an earthquake-caused rupture:* ***a.*** *diagram of the rupture and the mechanical model of the upper strata,* ***b.*** *acceptable scheme of horizontal deformation of the vertical section along the O–O' line,* ***c.*** *the case of a homogenous near-surface thickness, H - total thickness of the surface layer, Δ - distance from the rupture line to the observation point, v_{max} - velocity of the blocks at the rupture, $v(M,\Delta)$ - velocity of the blocks at the distance Δ from the rupture, $u(z,t)$ - function of displacement in perpendicular direction relative to the rupture, ρ_k, G_k, H_k - density, shear modulus and thickness of the k-th layer, $H + H_0$ - depth of the rupture. Arrows indicate the block's movement direction after rupture, the heavy line shows the rupture direction, dotted arrows show the direction of the layers' inertia movements after rupture (medium compression and dilatation)*

The method uses derived theoretical formulas to calculate all three parameters of the ground motion: displacements, velocities and accelerations during strong (with a magnitude of $M \geq 6.0$) earthquakes for any non-homogeneous (multilayer) ground beddings with various physical and mechanical characteristics – thicknesses, densities and shear moduli and at certain distance of Δ from the expected earthquake's rupture.

For homogeneous (single-layer) foundation beddings, all typical parameters of the above-mentioned values and their changes over time (seismograms and accelerograms) for an earthquake with magnitude of $6.0 \leq M \leq 9.0$ were derived and found to be nearly identical to the same parameters of actual earthquakes. Earthquake response spectra derived from synthetic accelerograms were also quite similar to the response spectra obtained through actual earthquake accelerograms [10]. This study addresses making synthetic seismograms and accelerograms for two-layer heterogeneous foundation beddings.

For the general case, the functions of the foundation bedding layers' inertia movement $u_k(z,t)$ after an earthquake-caused rupture must match the following wave equations [4,11]:

$$G_k \frac{\partial^2 u_k}{\partial z^2} - \rho_k \frac{\partial^2 u_k}{\partial t^2} + \eta_k \frac{\partial^3 u_k}{\partial z^2 \partial t} = 0, \quad k = 1,2...n \tag{1}$$

where: G_k - the shear modulus of the k-th layer of the rocks, ρ_k - its density, η_k - the coefficient of viscosity, H_k - thickness of the k-th layer, n - the number of layers. The value of initial velocity for the inertial movement of layers (which is equal to the after-rupture plate velocity, but in the opposite direction) is assumed as per the following formula [4]:

$$v(M,\Delta) = 100 \cdot e^{M-8.5}\left(1 - \frac{\Delta^2}{R^2}\right), \tag{2}$$

which represents the initial velocity (in cm/sec) of soil particles' vibration at future rupture ($\Delta = 0$), or at a distance of Δ from it, where R is the length of the deformed area measured from the expected earthquake rupture (outside the area of R, the medium is assumed undeformed by the design earthquake). The values of the length R and average slip $\bar{u}$ (in meters) depending on the magnitude M are determined per the following formulas [10]:

$$R = (5\bar{u} + 15) \times 10^3, \quad \lg \bar{u} = 0.55M - 3.71. \tag{3}$$

The summary of the values for $v(M,\Delta)$, R and $\bar{u}$ are provided in Table 1, along with rock stratum ultimate (limit) shear strain values $\gamma_{\lim}$, which were determined by the new methodology developed in [10], according to which the relationship between $\gamma_{\lim}$ and earthquake magnitude M is as follows:

$$10^4 \gamma_{lim} = 0.39M - 2.23. \tag{4}$$

By assuming the solutions of the differential equation (1) as

$$u_k(z,t) = \sum_{i=1}^{\infty} U_{ki}(z) q_i(t), \qquad h_{k-1} \le U_{ki}(z) \le h_k, \qquad k = 1,2...n, \tag{5}$$

and satisfying the boundary and initial conditions of the problem [3], the following transcendental equation for multilayer system, free oscillations frequencies and values of functions $U_{ki}(z)$ and $q_i(t)$ are obtained. The final values of displacements (seismograms) and accelerations (accelerograms) at the ground surface (z=0) would be as follows [3]:

$$\begin{aligned} u_1(0,t) &= 100 \cdot e^{M-8.5}\left(1 - \frac{\Delta^2}{R^2}\right) \sum_{i=1}^{\infty} U_{1i}(0) \frac{T_{0i}}{2\pi} \delta_i e^{-\frac{\Theta}{T_{0i}}t} \sin\frac{2\pi}{T_{0i}}t \\ u_1''(0,t) &= 100 \cdot e^{M-8.5}\left(1 - \frac{\Delta^2}{R^2}\right) \sum_{i=1}^{\infty} U_{1i}(0) \frac{2\pi}{T_{0i}} \delta_i e^{-\frac{\Theta}{T_{0i}}t} \sin\frac{2\pi}{T_{0i}}t \end{aligned}, \tag{6}$$

$$\delta_i = \frac{\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \rho_k U_{ki}(z) dz}{\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \rho_k U_{ki}^2(z) dz}, \quad i = 1,2,3... . \tag{7}$$

where: $p_i = \dfrac{2\pi}{T_{0i}}$ - the angular frequency of the whole multilayer system's free oscillations derived from the relevant transcendental frequency equation, n_i - the foundation bedding's attenuation coefficient for the i-th mode ($n_i^2 << p_i^2$, $\boldsymbol{\eta_k = \eta = const, k = 1, 2}$), T_{0i} - the period of the i-th mode of the foundation bedding particle's oscillation mode as shear seismic waves propagate with a velocity of (v_{Sk}) in the layers of a heterogeneous foundation bedding, and the functions $\boldsymbol{U_{ki}(z)}$ and $\boldsymbol{q_i(t)}$ are as follows:

$$\begin{aligned} &U_{ki}(z) = A_{ki} \sin \lambda_{ki} z + B_{ki} \cos \lambda_{ki} z \\ &q_i(t) = e^{-n_i t}\left(c_{1i} \sin p_i^* t + c_{2i} \cos p_i^* t\right), \ i = 1, 2,3... \\ &\frac{p_i^2 \rho_k}{G_k} = \lambda_{ki}^2, \ \frac{p_i^2 \eta}{G_k} = 2n_i, \qquad p_i^* = \sqrt{p_i^2 - n_i^2} \approx p_i \end{aligned} \tag{8}$$

Table 1. Values of the soil particles' oscillation velocities at the rupture $v(M,\Delta)$ in cm/sec, average slip $\overline{u}$ in meters, length of the deformed medium area R in km, and ultimate shear strain of ground γ_{lim} , depending on magnitude M and distance from the rupture Δ in km [10]

Magnitude M	**Average slip, $\overline{u}$ m**	**The length of deformed area measured from the rupture line, R, km**	**Rocks stratum ultimate (limit) shear strain values γ_{lim} x10^4 according to the formula (4)**	**Initial velocity at the rupture, v_{max}, cm/sec, ($\Delta = 0$)**	**Values of soil particles' oscillation velocities v (M,Δ) in cm/sec depending on magnitude M, and distance from the rupture ($\Delta < R$) in km[1]**																						
					Values of Δ, in km																						
					2	4	6	8	10	15	20 17 18 19	25 24	30 27	35 32	40 39	45	50 48	55	60 59	65 61	70	75	80	85	90	95	100 101
6	0.39	16.9	0.11	8.2	8.1	7.7	7.2	6.4	5.3	1.7																	
6.25	0.54	17.7	0.21	10.5	10.4	10.0	9.3	8.4	7.2	3.0	0.8																
6.5	0.73	18.6	0.31	13.5	13.4	12.9	12.1	11.0	9.6	4.7	0.9																
6.75	1.00	20	0.40	17.4	17.2	16.7	15.8	14.6	13.0	7.6	1.7																
7	1.38	21.9	0.50	22.3	22.1	21.6	20.6	19.3	17.7	11.8	3.7																
7.25	1.90	24.3	0.60	28.7	28.5	27.9	26.9	25.5	23.8	17.7	9.2	0.7															
7.5	2.60	28	0.70	36.8	36.6	36.0	35.1	33.8	32.1	26.2	18.0	7.5	2.6														
7.75	3.57	32.8	0.79	47.2	47.1	46.5	45.7	44.4	42.8	37.4	29.7	19.8	7.7	2.3													
8	4.90	39.5	0.89	60.7	60.5	60.0	59.3	58.2	56.8	51.9	45.1	36.4	25.7	13.0	1.5												
8.25	6.73	48.6	0.99	77.9	77.7	77.4	76.7	75.8	74.6	70.5	64.7	57.3	48.2	37.5	25.1	11.1	1.9										
8.5	9.23	61.1	1.09	100.0	99.9	99.6	99.0	98.3	97.3	94.0	89.3	83.3	75.9	67.2	57.1	45.8	33.0	19.0	3.6	0.3							
8.75	12.68	78.4	1.18	128.4	128.3	128.1	127.7	127.1	126.3	123.7	120.0	115.3	109.6	102.8	95.0	86.1	76.2	65.2	53.2	40.1	26.0	10.9	1.3				
9	17.38	102	1.28	164.9	164.8	164.6	164.3	163.9	163.3	161.3	158.5	155.0	150.6	145.5	139.5	132.8	125.3	116.9	107.8	97.9	87.2	75.7	63.5	50.4	36.5	21.9	6.4 3.2

[1] The values of the oscillation velocities of soil particles $\boldsymbol{v}$ $(\boldsymbol{M},\Delta)$ for values $\Delta < \boldsymbol{R}$ that are not included in the main table are additionally indicated in green.

Eigenmodes of oscillations for a two-layer base

Creation of synthetic seismograms and accelerograms is addressed in detail below for a two-layer foundation bedding, the diagram of which is shown on Figure 2.

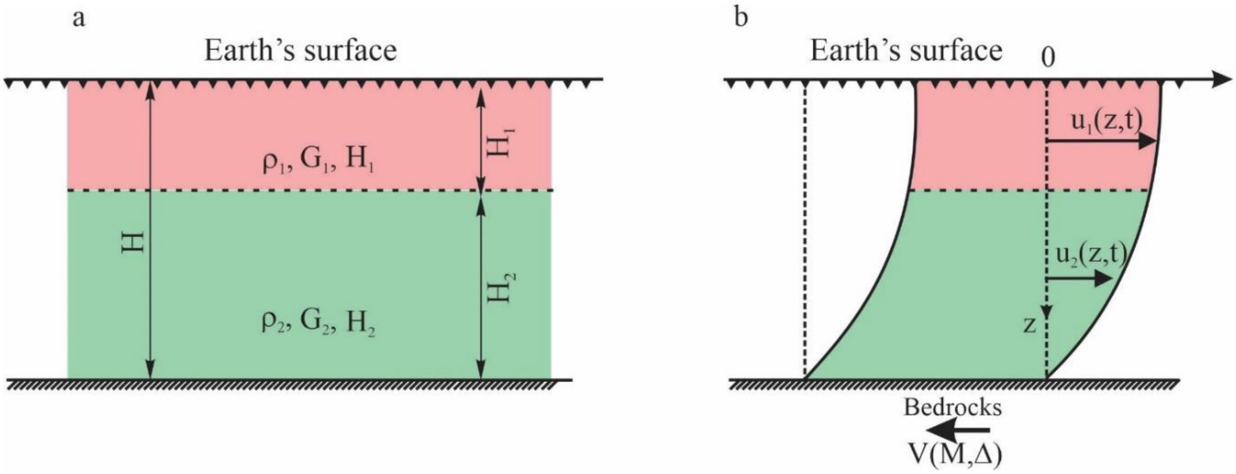


Fig. 2. *The diagram of a two-layer foundation bedding deformation*

The ground motion equation (1) for a two-layer foundation bedding is presented in the following forms:

$$G_1\frac{\partial^2 u_1}{\partial z^2}-\rho_1\frac{\partial^2 u_1}{\partial t^2}+\eta\frac{\partial^3 u_1}{\partial z^2\partial t}=0,\quad G_2\frac{\partial^2 u_2}{\partial z^2}-\rho_2\frac{\partial^2 u_2}{\partial t^2}+\eta\frac{\partial^3 u_2}{\partial z^2\partial t}=0\,. \tag{9}$$

Solutions of these equations by separation of variables can be presented as follows:

$$\begin{aligned} u_1(z,t)&=\sum_{i=1}^{\infty}U_{1i}(z)q_i(t)\quad \text{for}\quad 0\le z\le H_1\\ u_2(z,t)&=\sum_{i=1}^{\infty}U_{2i}(z)q_i(t)\quad \text{for}\quad H_1\le z\le H_1+H_2 \end{aligned}\,. \tag{10}$$

Inserting (10) and (9) and separating variables for the functions U_{1i} and U_{2i}, as well as functions $q_i(t)$ the following equations are obtained:

$$\begin{aligned} &U''_{1i}(z)+\lambda_{1i}^2U_{1i}(z)=0,\ U''_{2i}(z)+\lambda_{2i}^2U_{2i}(z)=0,\quad \lambda_{1i}^2=\frac{p_i^2\rho_1}{G_1},\ \lambda_{2i}^2=\frac{p_i^2\rho_2}{G_2}\\ &q''_i(t)+2n_iq'_i(t)+p_i^2q_i(t)=0,\quad \frac{\eta p_i^2}{G_k}=2n_i,\quad (i=1,2,3...) \end{aligned}\,. \tag{11}$$

Solutions of the equations (11) are then presented in the following form:

$$\begin{aligned} &U_{1i}(z)=A_{1i}\sin\lambda_{1i}z+B_{1i}\cos\lambda_{1i}z,\\ &U_{2i}(z)=A_{2i}\sin\lambda_{2i}z+B_{2i}\cos\lambda_{2i}z\\ &q_i(t)=e^{-n_it}(c_{1i}\sin p_it+c_{2i}\sin p_it),\ i=1,2,3... \end{aligned}\,, \tag{12}$$

where n_i is the layers' attenuation coefficient for the i-th mode.

Satisfying the boundary conditions of medium continuity at the layer separation and the initial conditions which are as follows:

$$\begin{aligned} &\text{boundary and medium continuity condition:}\\ &\text{for } z=0\quad G_1\frac{\partial u_1(z,t)}{\partial z}=0,\ \text{for } z=H_1+H_2\quad u_2(z,t)=0\\ &\text{for } z=H_1\quad u_1(z,t)=u_2(z,t),\quad G_1\frac{\partial u_1(z,t)}{\partial z}=G_2\frac{\partial u_2(z,t)}{\partial z}\\ &\text{initial conditions:}\\ &\text{for } t=0\quad u_k(z,t)=0,\ k=1,2,\\ &\text{for } t=0\quad \frac{\partial u_k(z,t)}{\partial t}=v(M.\Delta),\ k=1,2 \end{aligned}\,. \tag{13}$$

From the first four homogeneous equations (13) with respect to A_{1i}, A_{2i}, B_{1i}, B_{2i} (from equality to zero of the 4th-order determinant) the following transcendental equation is derived for determining frequencies $p_i = \frac{2\pi}{T_{0i}}$ [4]:

$$\sqrt{\frac{\rho_1 G_1}{\rho_2 G_2}}\, tg\frac{2\pi H_1}{T_{0i}}\sqrt{\frac{\rho_1}{G_1}} \cdot tg\frac{2\pi H_2}{T_{0i}}\sqrt{\frac{\rho_2}{G_2}} - 1 = 0 . \tag{14}$$

The values of coefficients A_{1i}, A_{2i}, B_{1i}, B_{2i} have the following form:

$$\begin{aligned} &A_{1i} = 0 \\ &A_{2i} = -B_{1i}\frac{\cos\lambda_1 H_1}{\sin\lambda_2 H_2}\cos\lambda_2 (H_1 + H_2), \qquad U_{1i}(z) = B_{1i}\cos\frac{2\pi z}{T_{0i} v_{S1}} \\ &B_{2i} = B_{1i}\frac{\cos\lambda_1 H_1}{\sin\lambda_2 H_2}\sin\lambda_2 (H_1 + H_2), \qquad U_{2i}(z) = -B_{1i}\frac{\sin\lambda_1 H_1}{\cos\lambda_2 H_2}\sin\frac{2\pi(z-H)}{T_{0i} v_{S2}} \end{aligned} . \tag{15}$$

From the last two equations (13) for coefficients c_{1i}, and c_{2i} we obtain [4,7,8]:

$$c_{2i} = 0, \qquad c_{1i} = \frac{\mathrm{v}(M,\Delta)\left(\int_0^{H_1}\rho_1 U_{1i}(z)dz + \int_{H_1}^{H_1+H_2}\rho_2 U_{2i}(z)dz\right)}{p_i\left(\int_0^{H_1}\rho_1 U_{1i}^2(z)dz + \int_{H_1}^{H_1+H_2}\rho_2 U_{2i}^2(z)dz\right)}, \quad i = 1, 2, 3 \ldots \tag{16}$$

where $T_{0i} = 2\pi/\rho_i$ is the free oscillations period of the *i*-th mode for a two-layer foundation bedding, the values of which are determined by the transcendental equation (14).

The final expressions for displacements and accelerations at the ground surface (i.e. *z*=0 and assuming that the attenuation coefficient values n_i are small) will be:

$$\begin{aligned} u_1(0,t) &= 100e^{M-8.5}\left(1-\frac{\Delta^2}{R^2}\right)\sum_{i=1}^{\infty} U_{1i}(0)\frac{T_{0i}}{2\pi}\delta_i e^{-\frac{\Theta_i}{T_{0i}}t}\sin\frac{2\pi}{T_{0i}}t \\ u_1''(0,t) &= 100e^{M-8.5}\left(1-\frac{\Delta^2}{R^2}\right)\sum_{i=1}^{\infty} U_{1i}(0)\frac{2\pi}{T_{0i}}\delta_i e^{-\frac{\Theta_i}{T_{0i}}t}\sin\frac{2\pi}{T_{0i}}t \end{aligned} , \tag{17}$$

where δ_i and Θ are designated. Θ is the decrement of vibrations of rock particles, which is assumed to be the same for all oscillation modes of the base.

$$\delta_i = \frac{\rho_1\int_0^{H_1} U_{1i}(z)dz + \rho_2\int_{H_1}^{H_1+H_2} U_{2i}(z)dz}{\rho_1\int_0^{H_1} U_{1i}^2(z)dz + \rho_2\int_{H_1}^{H_1+H_2} U_{2i}^2(z)dz}, \quad i = 1, 2, 3\ldots . \tag{18}$$

Inserting the values of $U_{1i}(z)$ and $U_{2i}(z)$ from (15) into (18), δ_i is derived as follows:

$$\delta_i = 4\cdot\frac{\rho_1\frac{v_{S1}}{v_{S2}}\sin\frac{2\pi H_1}{T_{0i}v_{S1}} + \rho_2\frac{\cos\frac{2\pi H_1}{T_{0i}v_{S1}}}{\sin\frac{2\pi H_2}{T_{0i}v_{S2}}}\cdot\left(1-\cos\frac{2\pi H_2}{T_{0i}v_{S2}}\right)}{B_{1i}\left[\rho_1\frac{v_{S1}}{v_{S2}}\left(\frac{4\pi H_1}{T_{0i}v_{S1}} + \sin\frac{4\pi H_1}{T_{0i}v_{S1}}\right) + \rho_2\left(\frac{\cos\frac{2\pi H_1}{T_{0i}v_{S1}}}{\sin\frac{2\pi H_2}{T_{0i}v_{S2}}}\right)^2\left(\frac{4\pi H_2}{T_{0i}v_{S2}} - \sin\frac{4\pi H_2}{T_{0i}v_{S2}}\right)\right]}, \quad i = 1, 2, 3\ldots . \tag{19}$$

Values of the constant B_{1i} can be assumed to be equal to 1 ($B_{1i} = 1$), because considering formulas (15) and (19), the final forms of (17) for seismograms and accelerograms $u_1(0,t)$ and $u_1''(0,t)$, the values of B_{1i} will cancel out.

Accelerograms of two-layer and their equivalent single-layer foundations on the Earth's surface

The 16 variants of heterogeneous foundation beddings are considered: the main parameters are presented in Table 2 and Figure 3, where H_1 and H_2, v_{S1} and v_{S2}, ρ_1 and ρ_2 are thicknesses, propagation velocities of seismic shear waves and ground layer densities, respectively, while free oscillation periods T_{01}, T_{02} and T_{03} for all 16 heterogeneous beddings are derived from the transcendental equations of frequencies (14), and values of coefficients δ_i are obtained from the formula (19).

Table 2. Physical and mechanical properties of heterogeneous foundation beddings

Heterogeneous foundation beddings

№	V_{s1}, cm/s	H_1, m	ρ_1, t*s²/m⁴	V_{s2}, cm/s	H_2, m	ρ_2, t*s²/m⁴	Derived from the formula (14)			According to the formula (19)		
							T_{01}, s	T_{02}, s	T_{03}, s	δ_1	δ_2	δ_3
1.	400	10	0.2	600	20	0.22	0.207	0.073	0.047	1.891	-0.522	0.483
2.	250	10	0.2	600	20	0.22	0.232	0.107	0.056	1.865	-0.873	0.861
3.	200	10	0.2	400	20	0.22	0.380	0.151	0.080	1.723	-0.701	0.714
4.	150	10	0.2	300	20	0.2	0.458	0.190	0.100	1.842	-0.787	0.498
5.	150	15	0.2	250	15	0.22	0.657	0.191	0.127	2.095	-1.000	0.704
6.	125	15	0.2	200	15	0.2	0.925	0.308	0.154	1.903	-0.488	0.486
7.	100	5	0.18	600	25	0.22	0.246	0.145	0.069	1.256	-1.104	0.867
8.	600	25	0.22	100	5	0.18	0.740	0.107	0.079	6.329	-0.126	0.330
9.	530	13	0.22	600	17	0.22	0.203	0.071	0.042	2.250	-0.612	0.465
10.	620	14	0.22	700	16	0.22	0.174	0.061	0.036	2.406	-0.631	0.510
11.	340	18	0.2	400	12	0.2	0.245	0.098	0.063	3.491	-1.105	0.346
12.	450	14	0.22	550	16	0.22	0.336	0.074	0.054	1.789	-0.672	0.346
13.	280	16.5	0.2	350	13.5	0.22	0.365	0.132	0.078	2.729	-0.813	0.688
14.	150	15	0.2	200	15	0.22	0.667	0.222	0.138	2.442	-0.816	0.623
15.	208	20	0.2	150	12	0.2	0.785	0.228	0.143	3.282	-0.587	0.443
16.	200	4.8	0.1626	300	30.7	0.1775	0.469	0.160	0.100	1.241	-0.604	0.437

Equivalent homogeneous beddings

№	V_{s1}, cm/s	H_1, m	ρ_1, t*s²/m⁴	V_{s2}, cm/s	H_2, m	ρ_2, t*s²/m⁴	$\bar{V}_s$, cm/s	$\bar{T}_{01}$, s	$\bar{T}_{02}$, s	$\bar{T}_{03}$, s	$\bar{\delta}_1$	$\bar{\delta}_2$	$\bar{\delta}_3$
1.	400	10	0.2	600	20	0.22	514	0.233	0.078	0.047	1.274	-0.425	0.255
2.	250	10	0.2	600	20	0.22	409	0.293	0.098	0.059	1.274	-0.425	0.255
3.	200	10	0.2	400	20	0.22	300	0.400	0.133	0.080	1.274	-0.425	0.255
4.	150	10	0.2	300	20	0.2	225	0.533	0.178	0.107	1.274	-0.425	0.255
5.	150	15	0.2	250	15	0.22	187	0.640	0.213	0.128	1.274	-0.425	0.255
6.	125	15	0.2	200	15	0.2	153	0.780	0.260	0.156	1.274	-0.425	0.255
7.	100	5	0.18	600	25	0.22	327	0.367	0.122	0.073	1.274	-0.425	0.255
8.	600	25	0.22	100	5	0.18	327	0.367	0.122	0.073	1.274	-0.425	0.255
9.	530	13	0.22	600	17	0.22	567	0.211	0.070	0.042	1.274	-0.425	0.255
10.	620	14	0.22	700	16	0.22	660	0.182	0.061	0.036	1.274	-0.425	0.255
11.	340	18	0.2	400	12	0.2	361	0.332	0.111	0.066	1.274	-0.425	0.255
12.	450	14	0.22	550	16	0.22	498	0.241	0.080	0.048	1.274	-0.425	0.255
13.	280	16.5	0.2	350	13.5	0.22	307	0.390	0.130	0.078	1.274	-0.425	0.255
14.	150	15	0.2	200	15	0.22	171	0.700	0.233	0.140	1.274	-0.425	0.255
15.	208	20	0.2	150	12	0.2	181	0.705	0.235	0.141	1.274	-0.425	0.255
16.	200	4.8	0.1626	300	30.7	0.1775	281	0.505	0.168	0.101	1.274	-0.425	0.255

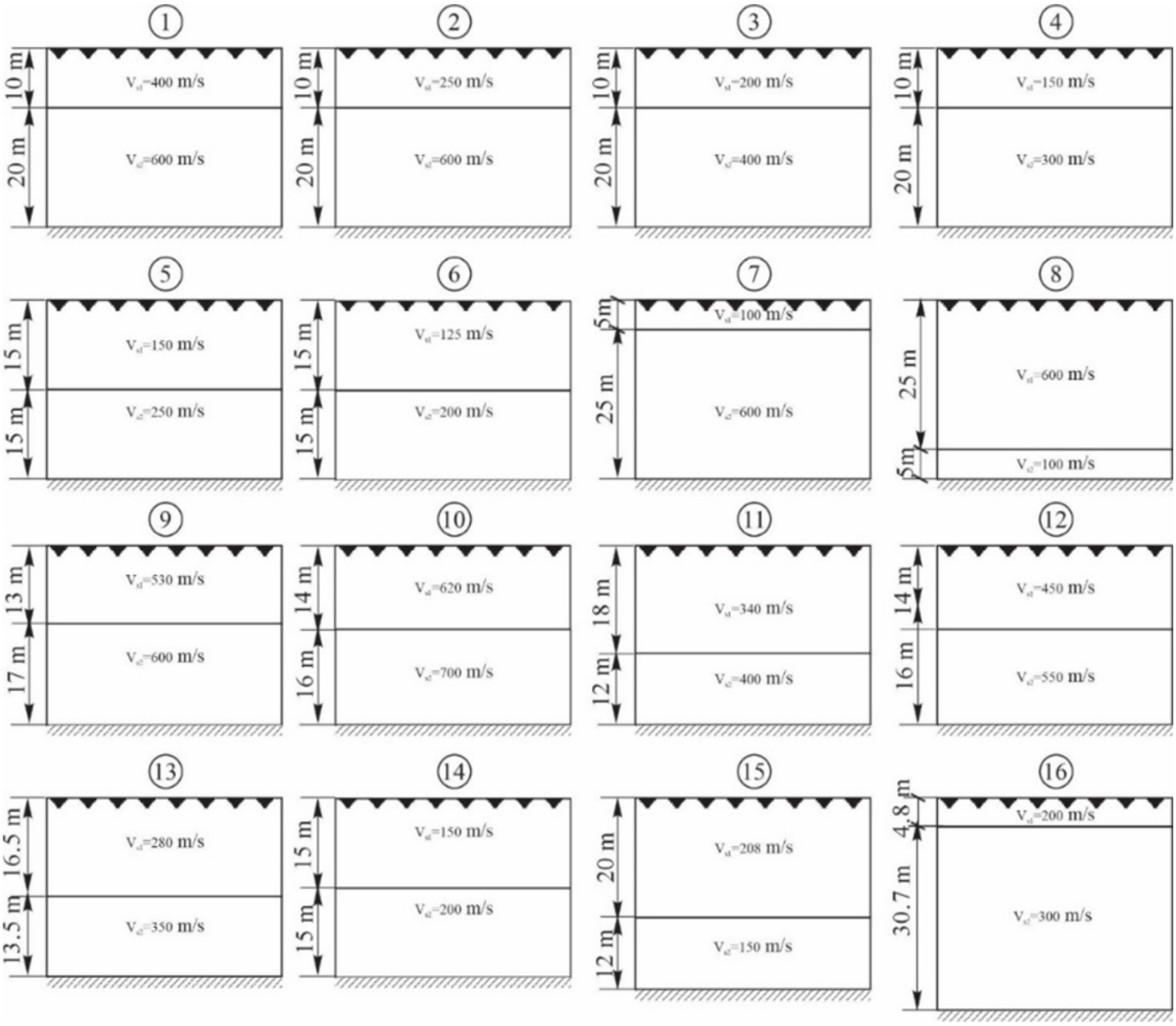


Fig. 3. *Diagrams of the considered heterogeneous foundation beddings*

Table 2 also provides the physical and mechanical characteristics for all equivalent homogeneous foundation beddings: average values of propagation velocity of displacement waves $\bar{v}_S$ and densities $\bar{\rho}$, which are determined by the following formulas[2].

$$\bar{v}_S = \frac{H_1 + H_2}{\dfrac{H_1}{v_{s1}} + \dfrac{H_2}{v_{s2}}}, \quad \bar{\rho} = \frac{H_1\rho_1 + H_2\rho_2}{H_1 + H_2}. \tag{20}$$

Values of the equivalent homogeneous foundation bedding first period are determined by the following formula:

$$\bar{T}_0 = 4(H_1 + H_2)/\bar{v}_s\,, \tag{21}$$

whereas values of the next periods $\bar{T}_{0i}$ (i=2,3,4…) and coefficients $\bar{\delta}_i$ (i=1,2,3…) are assumed by the formulas for a homogeneous foundation bedding [4,10]:

$$\bar{T}_{0i} = \frac{\bar{T}_{01}}{2i-1},\ (i=2,3,4\ldots), \quad \bar{\delta}_i = \frac{4}{\pi}\cdot\frac{(-1)^{i+1}}{2i-1}, \qquad i=1,2,3\ldots\,. \tag{22}$$

[2] RABC 20.04-2020. Earthquake Resistant Construction. Design Codes. Yerevan, 2020.

The final results of calculations according to formulas (17) for peak ground displacements and accelerations under M=7.0, Δ=15 km, and ground vibration decrement $\Theta_i = n_i T_{0i} = const\ (i = 1,2,3\ ...) = 0.3$ are provided in Table 3. As seen in the Table, when replacing heterogeneous bedding with an equivalent homogeneous one, the best approximation for ground accelerations u''_{0max} and $\overline{u}''_{0max}$ occurs when the accelerograms of the equivalent heterogeneous bedding are calculated factoring in five modes of oscillation, while for the heterogeneous bedding it is done in three modes.

Table 3. Peak values of seismograms and accelerograms for 16 heterogeneous beddings and equivalent homogeneous beddings

№	Heterogeneous beddings			Equivalent homogeneous bedding					Ratios of real displacements and accelerations of heterogeneous stratification taking into account **three** modes of oscillations to the **equivalent** taking into account **five** modes of oscillations	
	First oscillation mode period T_{01}, sec	Actual values of displacements and accelerations with consideration of **three** oscillations modes according to the formula (17)		Average values of the first oscillation mode period $\overline{T}_{01}$, sec	Equivalent values of displacements and accelerations with consideration of **three** oscillations modes according to the formula (17)		Equivalent values of displacements and accelerations with consideration of **five** oscillations modes according to the formula (17)			
		$u_{1max}(0)$, cm	$u''_{1max}(0)$, cm/s²		$\overline{u}_{1max}(0)$, sm	$\overline{u}''_{1max}(0)$, sm/s²	$\overline{u}_{1max}(0)$, sm	$\overline{u}''_{1max}(0)$, sm/s²	$\frac{u_{1max}(0)}{\overline{u}_{1max}(0)}$	$\frac{u''_{1max}(0)}{\overline{u}''_{1max}(0)}$
1	0.207	0.75	1482	0.233	0.58	965	0.591	1374	1.29	1.08
2	0.232	0.88	1714	0.293	0.73	768	0.745	1077	1.21	1.59
3	0.380	1.35	1008	0.400	1.00	572	1.021	833	1.35	1.21
4	0.458	1.70	737	0.533	1.34	428	1.361	617	1.27	1.19
5	0.657	2.79	782	0.640	1.61	357	1.634	520	1.73	1.50
6	0.925	3.35	370	0.780	1.96	292	1.991	421	1.71	0.88
7	0.246	0.63	1173	0.367	0.92	623	0.937	901	0.68	1.30
8	0.740	8.24	794	0.367	0.92	623	0.937	901	8.96	0.88
9	0.203	0.89	1799	0.211	0.53	1057	0.535	1460	1.68	1.23
10	0.174	0.80	1959	0.182	0.44	1085	0.444	1266	1.82	1.55
11	0.245	1.62	1796	0.332	0.83	682	0.846	970	1.95	1.85
12	0.336	1.08	1049	0.241	0.60	950	0.615	1383	1.80	0.76
13	0.365	1.95	1309	0.390	0.97	579	0.990	829	2.01	1.58
14	0.667	3.23	699	0.700	1.76	325	1.786	467	1.84	1.50
15	0.785	4.77	592	0.705	1.77	321	1.797	462	2.69	1.28
16	0.469	1.09	622	0.505	1.26	445	1.283	635	0.87	0.98
Average value excludes № 7 and 8[3]									1.66	1.3

Figure 4 shows superposition of the first three oscillation modes for heterogeneous (a) and equivalent homogeneous (b) foundation beddings. As it is apparent, although for the individual oscillation modes peak occurrences hardly differ in time, the resulting acceleration for the heterogeneous bedding on average is higher by 1.3 times compared to accelerations of the homogeneous bedding. Therefore, unlike with homogeneous bedding, the superposition process in heterogeneous one largely depends on coefficients δ_i, the values of which, unlike periods T_{0i}, depend on the medium deformation dynamic mode functions $U_{ki}(z)$ (k=1, 2). This is obvious also in Table 4, which provides comparative characteristics of the first oscillation modes for heterogeneous and equivalent homogeneous foundation beddings.

[3] These variants are excluded because the physical and mechanical properties of their layers significantly differ from each other, and therefore, the "method of averaging" is not applicable to them.

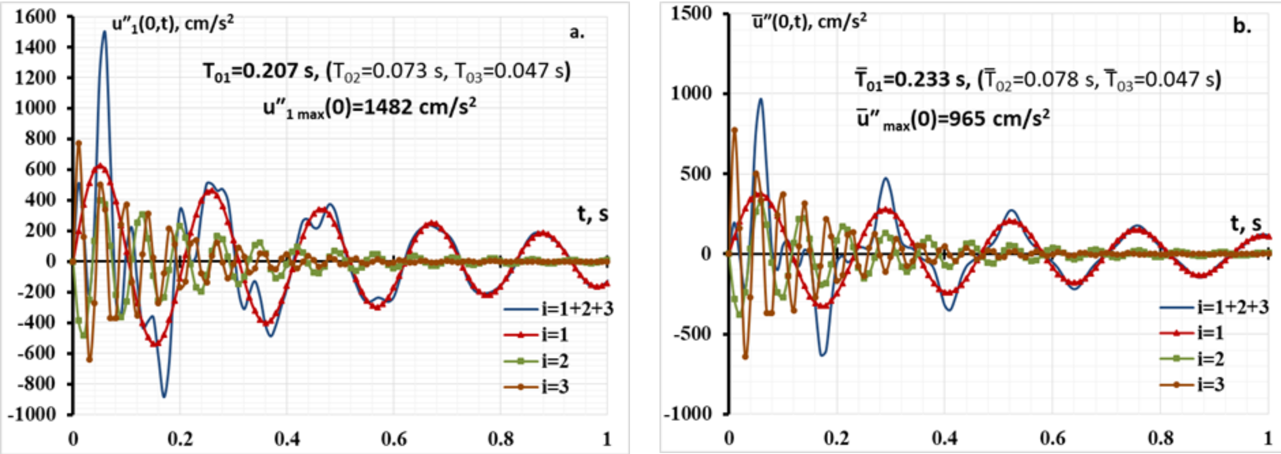


Fig. 4. *Acceleration time history curves (at ground surface) according to the formula (17): for individual oscillation modes (first i=1, second i=2, and third i=3), and their sum (i=1+2+3), for the № 1 variant of heterogeneous foundation bedding with* $T_{01} = 0.207$ *sec (a) and for the equivalent homogeneous bedding with average value* $\overline{T}_{01} = 0.233$ c. *(b), under: M=7.0, Δ=15 km,* $\Theta_1=\Theta_2=\Theta_3=0.3$

Table 4. Comparative characteristics of the fundamental oscillation periods and modes for heterogeneous and equivalent homogeneous foundation beddings

№	T_{01}, s	$\overline{T}_{01}$, s	$\frac{\overline{T}_{01}}{T_{01}}$	δ_1	δ_2	δ_3	$\frac{\delta_1}{\delta_2}$	$\frac{\delta_1}{\delta_3}$	$\overline{\delta}_1$ [4]	$\frac{\delta_1}{\overline{\delta}_1}$
1	0.207	0.233	1.127	1.890	-0.522	0.483	3.62	3.92	1.274	1.483
2	0.232	0.293	1.266	1.865	-0.873	0.861	2.14	2.17	1.274	1.464
3	0.380	0.400	1.053	1.723	-0.701	0.714	2.46	2.41	1.274	1.352
4	0.458	0.533	1.164	1.842	-0.787	0.498	2.34	3.70	1.274	1.446
5	0.657	0.640	0.974	2.095	-1.000	0.704	2.10	2.98	1.274	1.645
6	0.925	0.780	0.843	1.903	-0.488	0.486	3.90	3.91	1.274	1.494
7	0.246	0.367	1.492	1.256	-1.104	0.867	1.14	1.45	1.274	0.986
8	0.740	0.367	0.495	6.329	-0.126	0.330	50.07	19.15	1.274	4.968
9	0.203	0.211	1.042	2.250	-0.612	0.465	3.67	4.84	1.274	1.766
10	0.174	0.182	1.044	2.406	-0.631	0.510	3.81	4.72	1.274	1.889
11	0.245	0.332	1.354	3.491	-1.105	0.346	3.16	10.09	1.274	2.740
12	0.336	0.241	0.718	1.789	-0.672	0.346	2.66	5.17	1.274	1.404
13	0.365	0.390	1.069	2.729	-0.813	0.688	3.36	3.96	1.274	2.142
14	0.667	0.700	1.050	2.442	-0.816	0.623	2.99	3.92	1.274	1.917
15	0.785	0.705	0.898	3.282	-0.587	0.443	5.59	7.41	1.274	2.576
16	0.469	0.505	1.076	1.241	-0.604	0.437	2.05	2.84	1.274	0.974
Average value excludes № 7 and 8			1.05				3.1	4.4		1.73

Seismograms and accelerograms for the first 8 variants are shown in Figure 5.

[4] The ratio $\overline{\delta}_I/\overline{\delta}_i$ for any homogeneous bedding equals to $(-1)^{i+1}(2i-1)$, (i=1,2,3…).

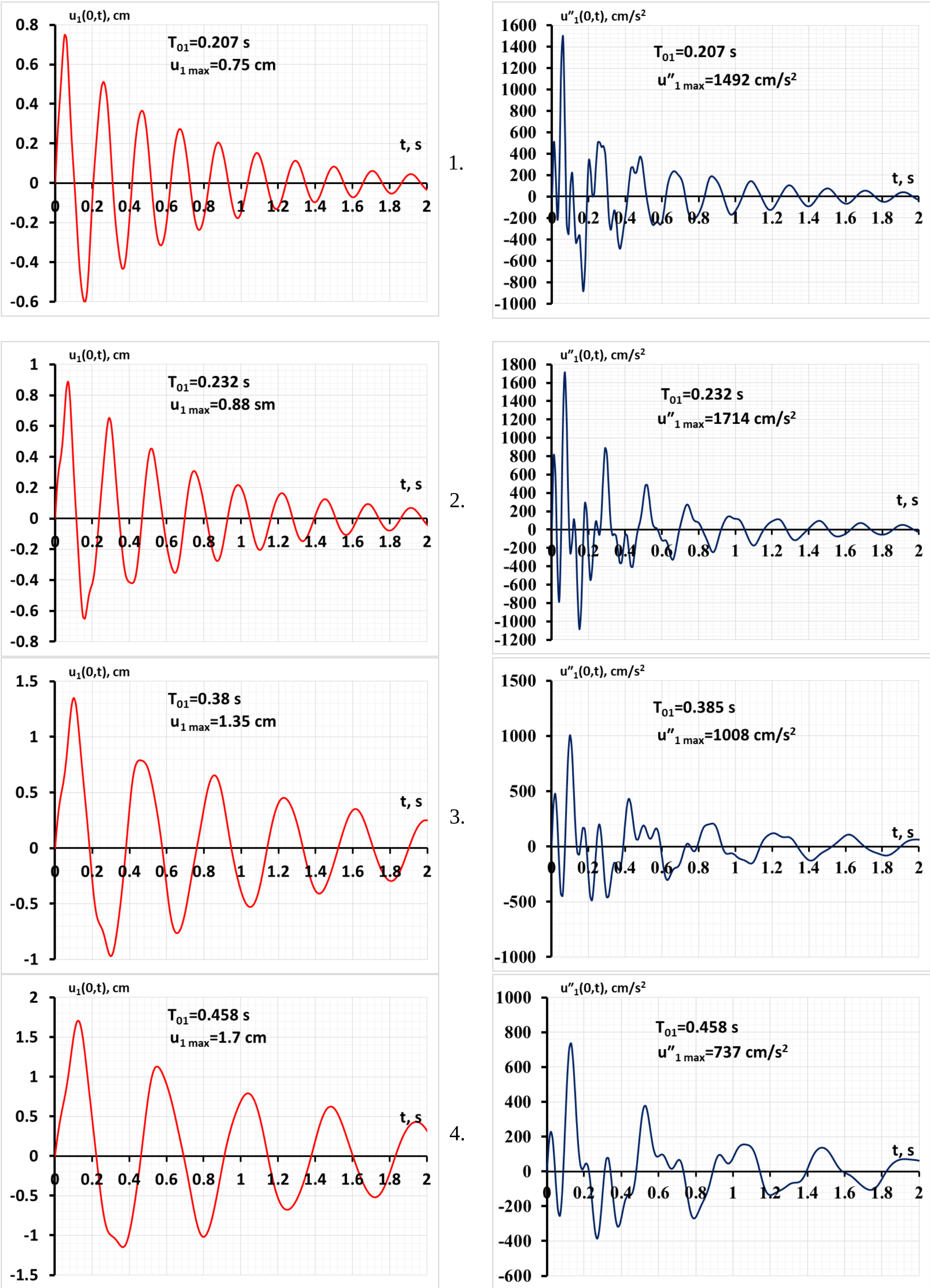
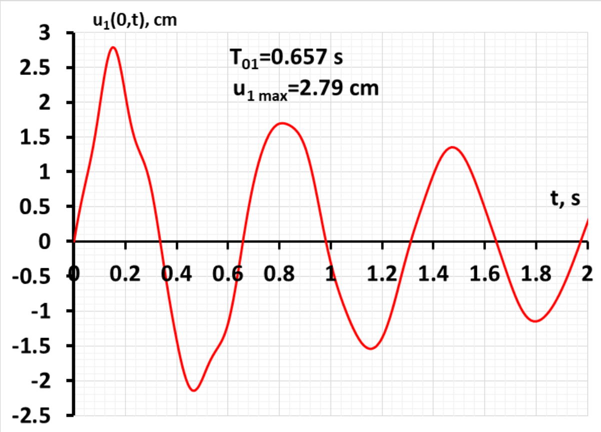


Fig. 5. *Synthetic seismograms and accelerograms of 16 variants of heterogeneous foundation beddings (Fig.3)*

Fig. 5. (continued)



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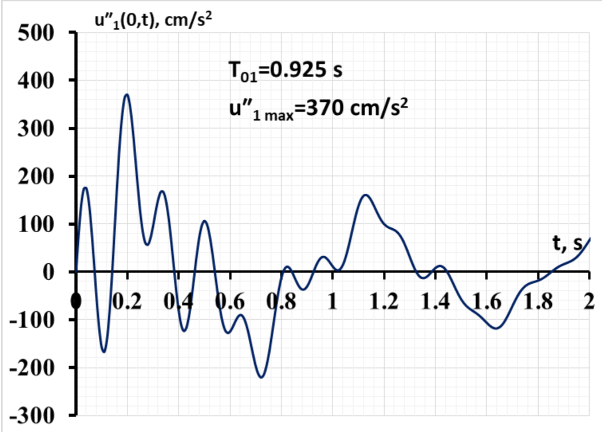

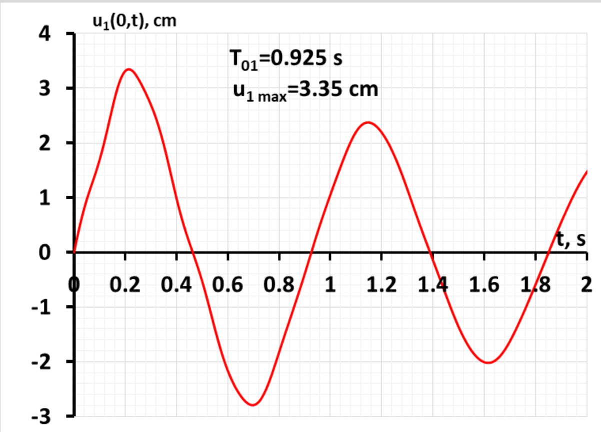

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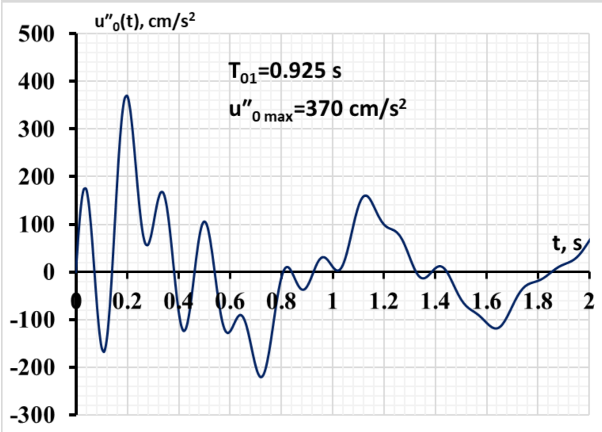

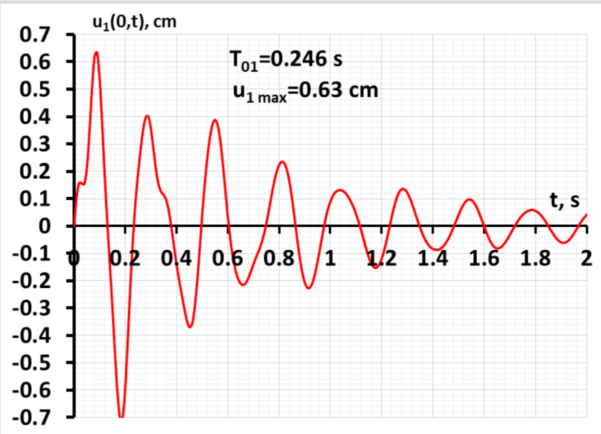

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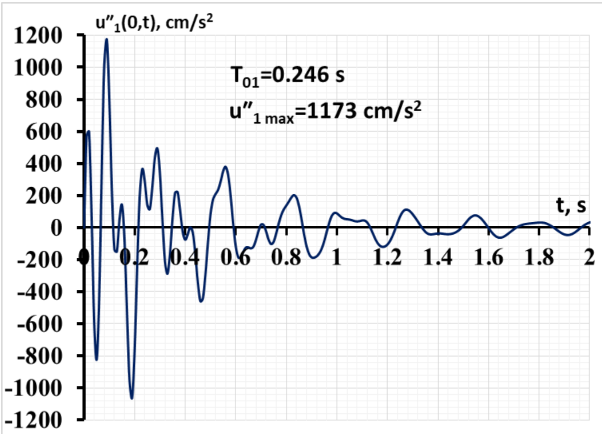

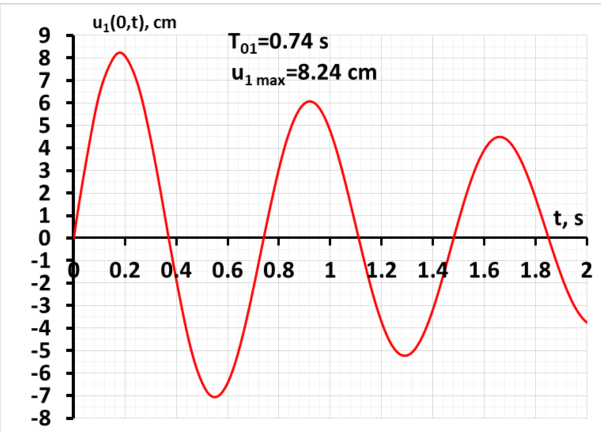

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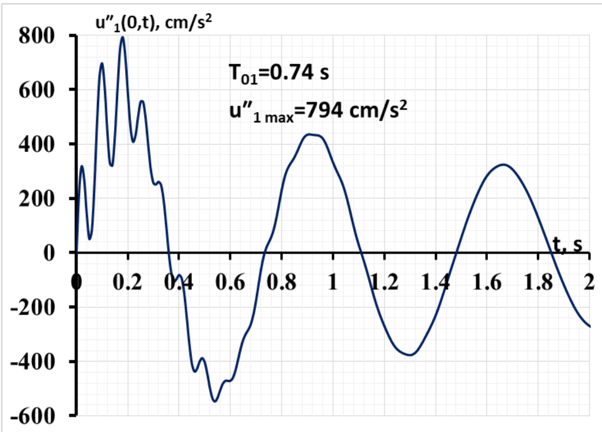
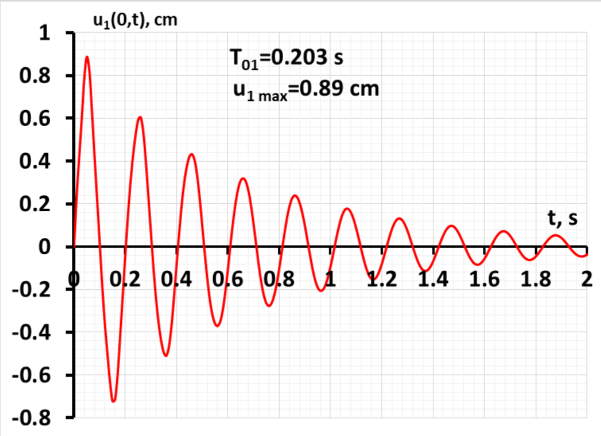


Fig. 5. (continued)



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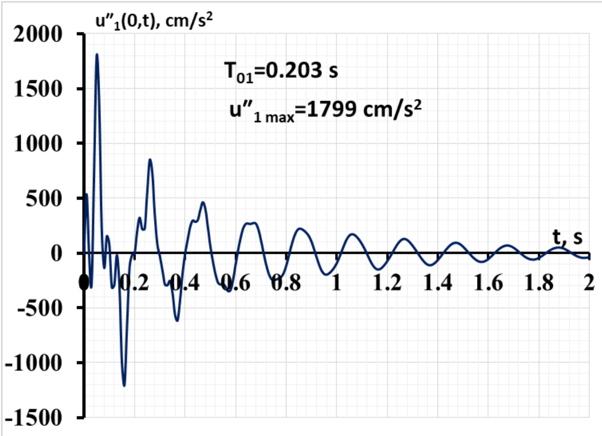

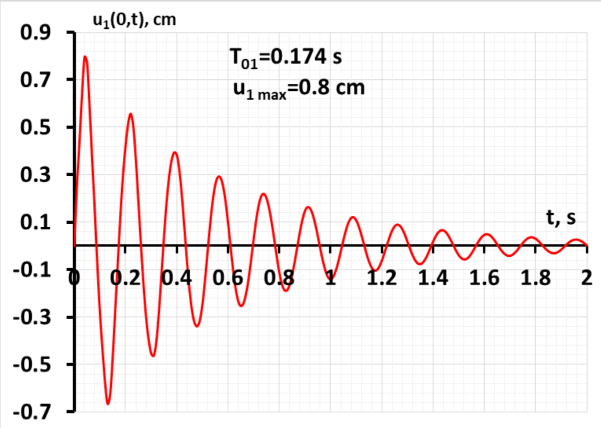

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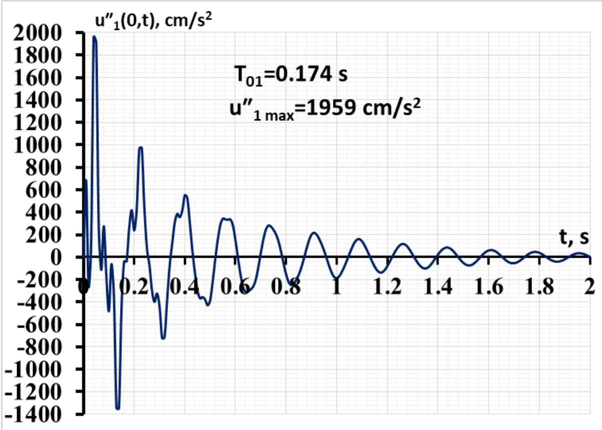

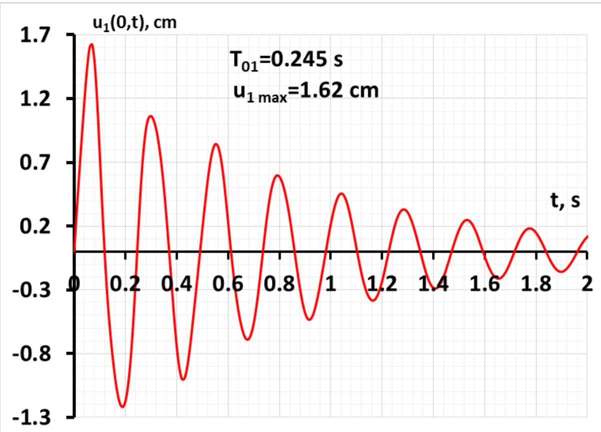

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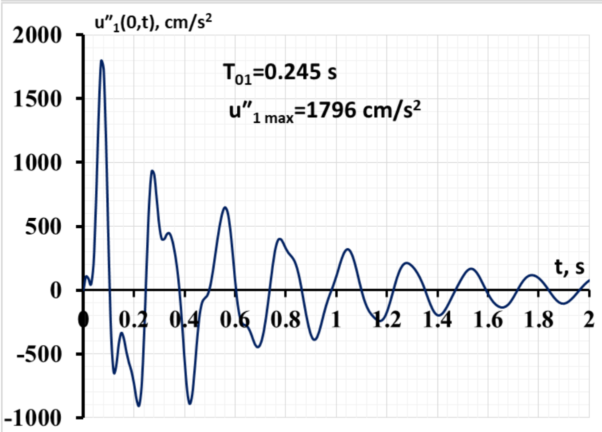

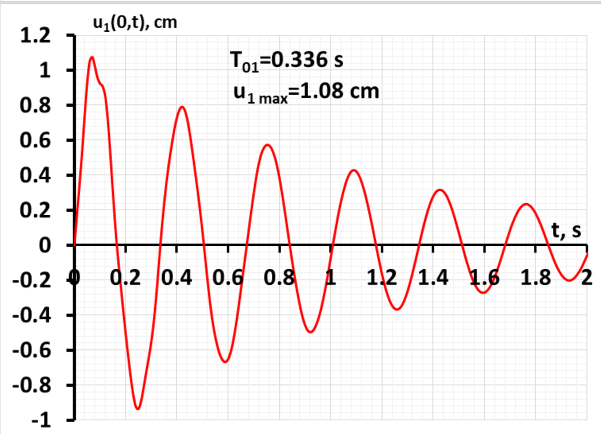

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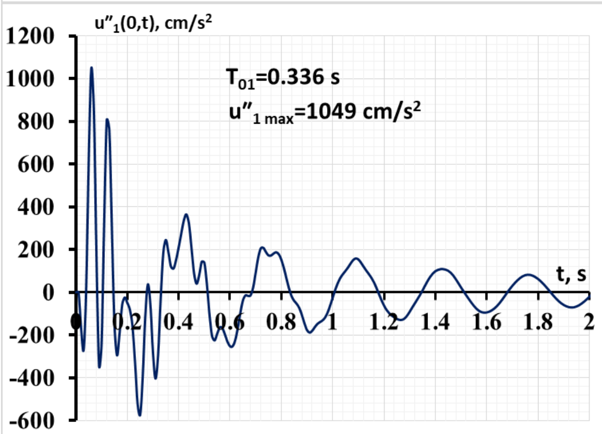

Fig. 5. (continued)

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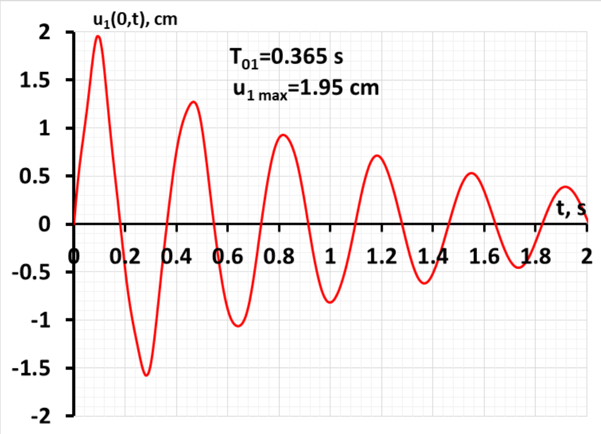

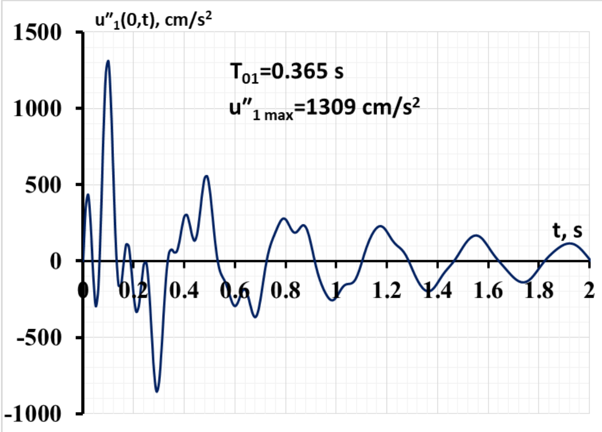

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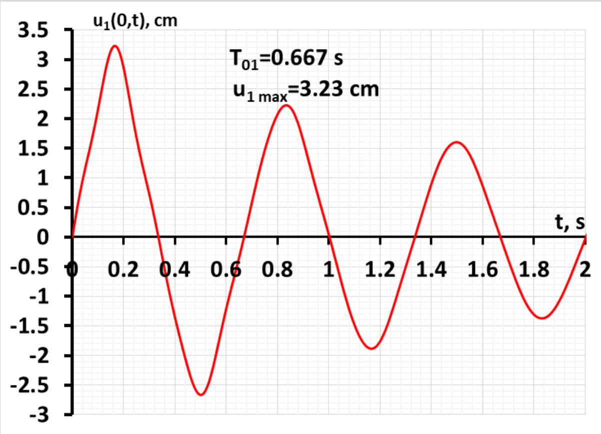

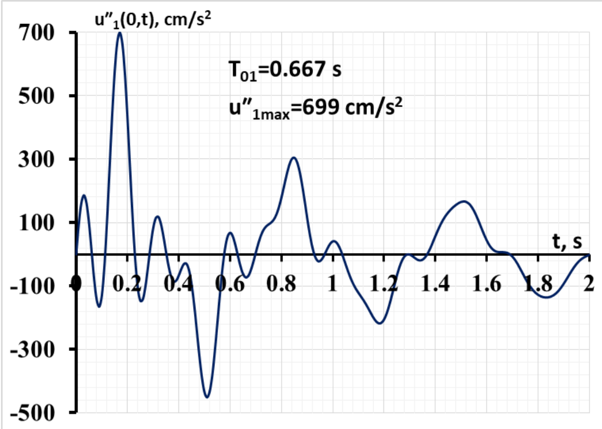

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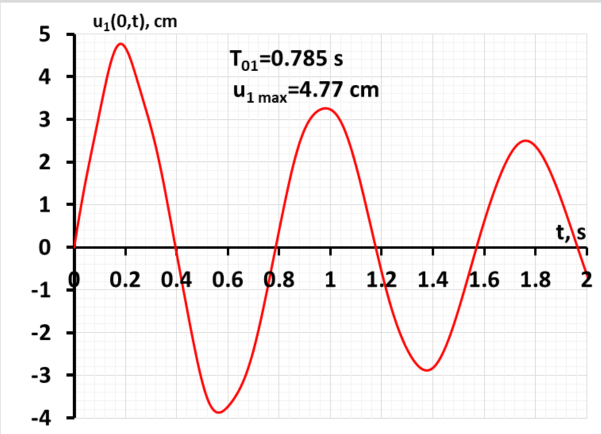

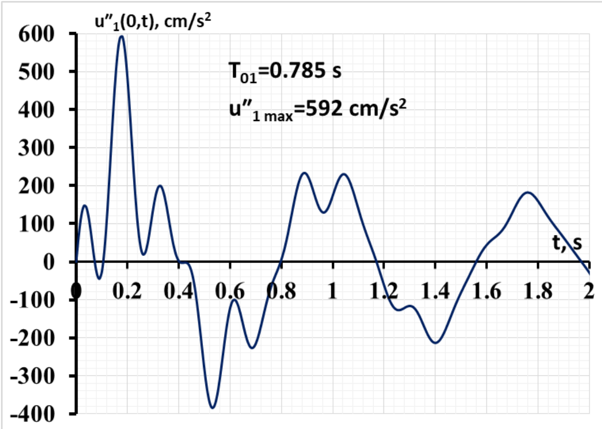

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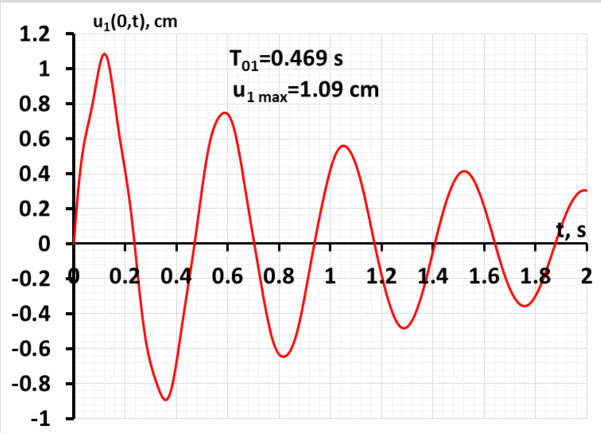

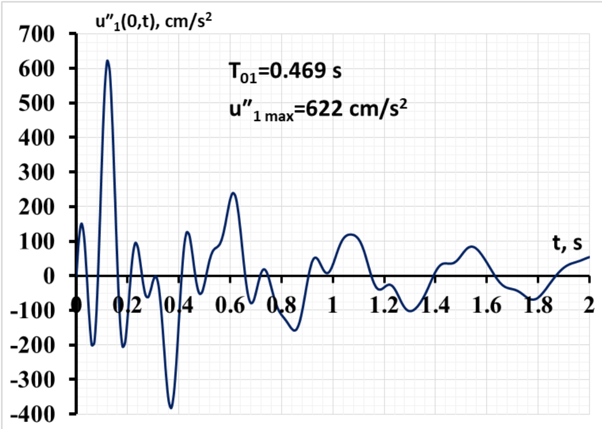

Conclusion

The article develops a method for predicting seismograms and accelerograms for heterogeneous foundation beddings with known physical and mechanical characteristics of the bedrock layers, depending on the earthquake magnitude M≥6.0 and the distance of a given construction site from the fault line. For 16 typical two-layer beddings in soils of various seismic categories, seismograms and accelerograms were obtained for M=7.0 and distance from rupture line Δ=15 km. A method is suggested to replace heterogeneous foundation beddings with equivalent homogeneous ones and quantitative estimates of such replacements are derived. It is shown that the fundamental oscillation mode period T_{01} of the actual heterogeneous bedding changes (both increases and decreases) on average by 1.5 times, whereas the second and third oscillation mode periods on average by 1.3, and 1.05 times, respectively, compared to the periods of the second and third oscillation modes of the heterogeneous bedding. It should be noted that with such replacements there could be an increase in ground accelerations up to 1.6 times. Calculations for the equivalent homogeneous bedding that consider five oscillation modes instead of three, show ground acceleration decreases up to 1.3 times. It has been found out that heterogeneity of the foundation bedding, in fact, leads to increased acceleration and displacement values at the ground surface due to a significant change in the modes of dynamic deformation of the medium (the function $U_{ki}(z)$ and coefficients δ_i) compared to the homogeneous beddings.

Conflict of Interest

The authors declare no conflicts of interest.

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