RESEARCH OF UNSTEADY TURBULENT MOTION IN A FLAT-PARALLEL PIPE

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Abstract: *This paper focuses on studying structural changes in the viscous fluid during turbulent unsteady plane-parallel pressure flow. This investigation analyzes how hydrodynamic parameters change in viscous fluid unsteady motion, particularly by calculating the turbulent viscosity coefficient. The study addresses the boundary problem that arises when there are axisymmetric changes in the flow. The selection of boundary conditions aligns with the patterns associated with the arbitrary distribution of pressure gradients and velocities within the section. Based on the initial and boundary conditions, the boundary value problem is formulated. The method for solving this boundary value problem was developed, and the regularities of the instantaneous speed change along the cross-section were obtained. The solution to the boundary value problem is derived by integrating partial differential equations, ensuring the satisfaction of all boundary conditions. Analytical solutions have been derived, enabling the determination of velocity patterns at any given moment. On the basis of the general solutions to the problem, solutions were obtained for the accelerating motion under the influence of a constant pressure gradient on a fluid at rest. The computer analysis generated composite graphs displaying average velocities across various time intervals. The provided solutions enable the visualization of average velocity changes within conditions of plane-parallel turbulent flow. These findings allow for the conclusion of the design of individual units within hydro-mechanical equipment.*

Keywords: *plane-parallel motion, turbulent motion, viscous fluid, velocity distribution.*

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Introduction

A characteristic feature of pressure systems is that ongoing hydro-mechanical phenomena are frequently accompanied by an unsteady flow of the working medium over time, resulting in changes in velocity and pressure at any point along the system.

When discussing practical problems, the one-dimensional unsteady motion model is employed as a computational framework to analyze these changes. In this model, current parameters are determined based on averaged values taken across the cross-section.

Kinetic energy coefficients and momentum take into account deviations of the averaged quantities from the hydro-mechanical parameters of the current shear points. Therefore, one of the issues of unsteady motion is the study of the change patterns of the mentioned coefficients, for which it is necessary to perform structural studies of unsteady motion.

Frequently, the necessity arises to employ complex mathematical models for conducting these studies. However, the results obtained from such models may not be suitable for practical applications. Therefore, to make certain simplifications in these calculations, we proceed from the quasi-stationary movement model. In this case, the unsteady flow is considered as a sequence of stationary motions.

At each moment in time, the average speed of the current is equated to the average speed of what appears to be stationary motion. Therefore, unsteady motion is conceptualized as a series of successive stationary motions. This assumption can yield realistic results, provided that the plot of the instantaneous velocity distribution closely adheres to the quadratic parabolic law.

However, theoretical and experimental research suggests that the graphs representing the instantaneous velocity distribution can actually diverge considerably from the quadratic parabolic law. This deviation suggests that the frictional stress near the stationary wall cannot be the only factor influencing energy losses, as assumed in the quasi-stationary motion framework.

The frictional stress near the wall varies depending on the law governing the change in instantaneous velocity within the section, causing it to differ from the corresponding frictional stress in the quasi-stationary motion. In unsteady motion, to determine the variation in frictional stress occurring near the stationary wall, it is essential to examine the pattern of velocity change within the effective cross-section.

Literature review and problem statement

Thus, the main goal of the study is to determine the structural changes in the pressure unsteady flow of viscous fluid depending on time and coordinates of the points of the effective cross-section [5,6].

The plane-parallel laminar flow is among the least common motions of this nature [7,8]. Laminar flow mainly occurs in machine tools in narrow rectangular shear paths, which are characteristic of lubricated bearings [6].

Plane-parallel flows are mainly in the turbulent regime, and the hydrodynamic quantities often change depending on time, i.e., they are unsteady. In this sense, the study of turbulent unsteady motion has important practical significance and theoretical interest.

During turbulent motion, frictional stresses arising between current layers depend on the distance from the stationary wall [1,9]. In a turbulent current, the solutions obtained by the function introduced for the calculation of the turbulent stresses depending on the distance from the stationary wall provide the findings of the structural changes of the current [10]. Similar research was conducted in [14].

In reference [15], an attempt was made to account for the velocity component normal to the wall in addition to the longitudinal velocity component. The novelty of this research lies in the inclusion of the second velocity component, which was made possible through thoroughly conducted experiments. According to the study results, not all values of turbulent stress approach the self-similar asymptotic state at the same rate as the Reynolds number increases, with the Reynolds shear stress approaching this state more rapidly than the longitudinal normal stress.

Reference [16] presents the primary accomplishments in the study of the transition period and turbulence over the past thirty years. Despite all efforts, some inevitable omissions will continue to be explored as turbulence research continues to deepen. The impact of the pressure gradient on the universal logarithmic law governing the average velocity profile was investigated in reference [17].

The Aim and objectives of the study

The research aims to uncover the patterns governing changes in the hydrodynamic parameters of a viscous fluid during unsteady turbulent plane-parallel flow.

To achieve this objective, the following tasks are addressed:

- formulating the boundary value problem and specifying the initial and boundary conditions,
- developing a method to solve the boundary value problem and unveiling the patterns of change in hydrodynamic parameters in unsteady turbulent plane-parallel flow,
- constructing graphs illustrating the variations in axial velocities over time and with changes in pressure gradient,
- determining the stabilization time of the unsteady process.

Materials and Methods

Choosing a Calculation Scheme

As is well known, frictional stresses develop between layers of fluid in turbulent flow, with their magnitude dependent on the position of the current point.

According to Boussinesq, frictional stresses occurring between fluid layers in the case of smooth lateral movement are determined by the y-coordinate measured from the stationary wall, as described by the following equations [1,2 and 9].

$$\tau = -\rho\varepsilon\frac{d\overline{u}}{dy}, \tag{1}$$

where ε is the kinematic coefficient of turbulent viscosity.

Kinematic viscous factor in case of turbulent unsteady flow has not been studied deeply, thoroughly. Therefore, to investigate turbulent unsteady flow, we will initially employ the assumptions made for stationary turbulent flows as a first approximation [4]. The kinematic coefficient of turbulent viscosity, depending on the distance of the point from the stationary wall, changes according to the linear law:

$$\varepsilon = n \cdot y. \tag{2}$$

A perfect match between theoretical and experimental investigations is provided by the link between the kinematic coefficients of turbulent viscosity (2) for stationary turbulent motions [2]. When stationary turbulent motion occurs under the flow section, this linear relationship results in a logarithmic power distribution. This power distribution closely matches the findings of experimental research [7].

Matching the linear relationship of the kinematic coefficient of viscosity in stationary turbulent flows with considerable accuracy can also provide some accuracy in unsteady turbulent motions.

Statement of the problem and formulation of the system of differential equations for the study

To formulate the equation of unsteady turbulent motion, let's isolate the mass of the elemental fluid from the moving current and establish the differential equation of its motion, so we have (Fig.1)

$$\rho\frac{\partial\overline{u}}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\partial\tau}{\partial y}. \tag{3}$$

Inserting the value of the kinematic coefficient of turbulent viscosity in (3) we get

$$\frac{\partial\overline{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + n\left(\frac{\partial\overline{u}}{\partial y} + y\frac{\partial^2\overline{u}}{\partial y^2}\right). \tag{4}$$

Assuming plane-parallel flow to be axisymmetric and isothermal, it follows that the pressures within each fixed cross-sectional area are the same across all its points, meaning that:

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial z} = f(t). \tag{5}$$

Therefore, the study of the unsteady turbulent motion in the case of a plane-parallel flow is presented by

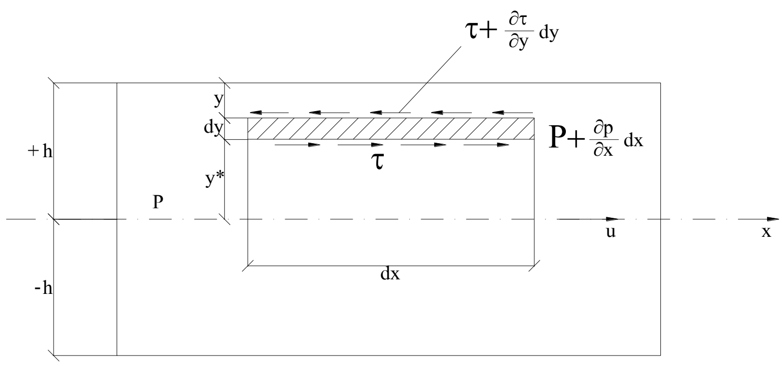

Fig. 1. *Schematic diagram of the plane-parallel turbulent flow*

$$\frac{\partial\overline{u}}{\partial t} = -f(t) + n\left(\frac{\partial\overline{u}}{\partial y} + y\frac{\partial^2\overline{u}}{\partial y^2}\right) \tag{6}$$

The above equation can be solved under the following boundary conditions:

$$\overline{u}(y,t) = 0, \quad \text{when} \quad y = 0, \quad t > 0,$$
$$\overline{u}(y,t) = \phi(y), \ \text{when} \quad t = 0, \quad (0 < y < 2h). \tag{7}$$

Let's import dimensionless variables:

$$U(x,t)=\frac{\bar{u}}{U_0},\quad \frac{y}{h}=x,\quad \frac{P}{P_0}=p\ , \tag{8}$$

where U_0 is the characteristic velocity of the cross-section, which is equal to the average velocity of the flow cross-section:

$$U_o=\frac{1}{h}\int_0^{+h}\varphi(y)dy. \tag{9}$$

Eq. (6) and the boundary conditions (8), when expressed in terms of dimensionless variables, take the following form:

$$\frac{\partial U}{\partial t}=-F(t)+\frac{n}{h}\left(\frac{\partial U}{\partial x}+x\frac{\partial^2 U}{\partial x^2}\right)\ , \tag{10}$$

$$U(x,t)=0,\quad \text{at}\quad x=0, t>0, \tag{11}$$

$$U(x,t)=\phi(x),\ \text{at}\quad t=0,\quad (0<x\le 1), \tag{12}$$

$$\frac{\partial U(x,t)}{\partial x}|x=1=0, \tag{13}$$

where

$$F(t)=\frac{P_0}{\rho U_0}\frac{\partial P}{\partial x}.$$

Let's look for the general solution of (10) in the following form [3]:

$$U(x,t)=\sum_{k=1}^{\infty}C_k(t)\cdot J_1\left(\lambda_k\sqrt{x}\right). \tag{14}$$

It follows from the boundary condition (13) that

$$\frac{\partial U(1,t)}{\partial x}=\sum_{k=1}^{\infty}C_k(t)\cdot J_1'(\lambda_k)=0,\ \text{where}\quad J_1'(\lambda_k)=0. \tag{15}$$

Hence, (15) represents the characteristic equation of the problem, and its roots will serve as the eigenvalues. Inserting the general solution (14) into (10), we get

$$\sum_{k=1}^{\infty}C_k'(t)\cdot J_1\left(\lambda_k\sqrt{x}\right)=\frac{n}{h}\sum_{k=1}^{\infty}C_k(t)\cdot\left[\lambda_k^2 xJ_1''\left(\lambda_k\sqrt{x}\right)+J_1'\left(\lambda_k\sqrt{x}\right)\right]-F(t). \tag{16}$$

It is known that the solution of the below equation

$$\lambda_k^2 xJ_1''\left(\lambda_k\sqrt{x}\right)+J_1'\left(\lambda_k\sqrt{x}\right)+\frac{1}{4}\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right)=0 \tag{17}$$

is the function $J_1\left(\lambda_k\sqrt{x}\right)$ [3], therefore (16) will be transformed into the following form:

$$\sum_{k=1}^{\infty}C_k'(t)\cdot J_1\left(\lambda_k\sqrt{x}\right)=\frac{n}{h}\sum_{k=1}^{\infty}C_k(t)\cdot\left[\frac{1}{4}\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k\sqrt{x}\right)\right]-F(t). \tag{18}$$

To solve the problem, let us expand the $F(t)$ function of the Fourier-Bessel series according to the eigenfunctions of the problem [3], we have

$$F(t)=\sum_{k=1}^{\infty}a_k\cdot J_1\left(\lambda_k\sqrt{x}\right), \tag{19}$$

where a_k are coefficients of the series.

In order to determine the above coefficients, multiply the two parts of (19) by the expression $J_1\left(\lambda_m \sqrt{x}\right)dx$ and integrate in the (0.1) range, we have

$$\int_0^1 F(t)\cdot J_1\left(\lambda_m \sqrt{x}\right)dx = \sum_{k=1}^{\infty} a_k \int_0^1 J_1\left(\lambda_k \sqrt{x}\right)\cdot J_1\left(\lambda_m \sqrt{x}\right)dx \ . \tag{20}$$

In the case of arbitrary λ_m and λ_k values, we have the following relations:

$$\int_o^1 J_1\left(\lambda_k \sqrt{x}\right)\cdot J_1\left(\lambda_m \sqrt{x}\right)dx = \begin{cases} 0 & \text{when } k \neq m \\ \int_0^1 J_1^2\left(\lambda_k \sqrt{x}\right)dx = \dfrac{\lambda_k^2 - 1}{\lambda_k^2}, & \text{when } k = m \end{cases} \tag{21}$$

From (19), (20) and (21) we get

$$a_k(t) = \frac{F(t)L_0(\lambda_k)}{L_1(\lambda_k)} \ , \tag{22}$$

where

$$L_0(\lambda_k) = \int_0^1 J_1\left(\lambda_k \sqrt{x}\right)dx \ , \tag{23}$$

$$L_1(\lambda_k) = \int_0^1 J_1^2\left(\lambda_k \sqrt{x}\right)dx \ . \tag{24}$$

Inserting the value of the function F(t) from the (19) into the (18), we get

$$\sum_{k=1}^{\infty} C_k'(t)\cdot J_1\left(\lambda_k \sqrt{x}\right) = \frac{n}{4h}\sum_{k=1}^{\infty} C_k(t)\cdot\left[\left(1-\frac{1}{\lambda_k^2 x}\right)J_1\left(\lambda_k \sqrt{x}\right)\right] - \sum_{k=1}^{\infty} a_k(t)\cdot J_1\left(\lambda_k \sqrt{x}\right). \tag{25}$$

To determine the $C_k(t)$ coefficients, let's multiply both parts of (25) by $J_1\left(\lambda_k \sqrt{x}\right)dx$ and integrate in the interval [0.1]. Taking into account the function orthogonality condition (21), we have

$$C_k'(t) + \frac{n}{4h}\frac{L_2(\lambda_k)}{L_1(\lambda_k)}C_k(t) = a_k(t), \tag{26}$$

where

$$L_2(\lambda_k) = \int_0^1 \left(1-\frac{1}{\lambda_k^2 x}\right)J_1^2\left(\lambda_k \sqrt{x}\right)dx \ . \tag{27}$$

Let's present (26) in the following form:

$$C_k'(t) + \alpha\beta\ C_k(t) = -a_k(t), \tag{28}$$

where

$$\alpha = \frac{n}{4h}, \quad \beta_k = \frac{L_2(\lambda_k)}{L_1(\lambda_k)}.$$

The solution of (28) will be

$$C_k(t) = \exp(-\alpha\beta_k t)\int_0^t a_k(\tau)\exp(\alpha\beta_k \tau)d\tau + C_k \exp(-\alpha\beta_k t) \ .$$

Inserting the value $a_k(t)$ into the last equation, we have

$$C_k(t) = \frac{L_0(\lambda_k)}{L_1(\lambda_k)}\exp(-\alpha\beta_k t)\int_0^t F(\tau)\exp(\alpha\beta_k \tau)d\tau + C_k \exp(-\alpha\beta_k t). \tag{29}$$

Let's consider a special case of the problem, when $F(\tau) = B_0 = Const$, we have

$$C_k(t)=\frac{L_0(\lambda_k)B_0}{\alpha\beta_k L_1(\lambda_k)}\left(1-\exp(-\alpha\beta_k t)\right)+C_k\exp(-\alpha\beta_k t). \tag{30}$$

Inserting the value $C_k(t)$ into (14) we have

$$U(x,t)=\sum_{k=1}^{\infty}\left\{C_k\exp(-\alpha\beta_k t)+\frac{L_0(\lambda_k)B_0}{\alpha\beta_k L_1(\lambda_k)}\left(1-\exp(-\alpha\beta_k t)\right)\right\}\cdot J_1\left(\lambda_k\sqrt{x}\right). \tag{31}$$

In accordance to the initial $U(x,0)=\varphi(x)$ condition of the problem (12), we get

$$\phi(x)=\sum_{k=1}^{\infty}C_k\cdot J_1\left(\lambda_k\sqrt{x}\right), \tag{32}$$

from where

$$C_k=\frac{\lambda_k^2 L_k(\lambda_k)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}, \tag{33}$$

$$L_k=\int_0^1\phi(x)J_1\left(\lambda_k\sqrt{x}\right)dx. \tag{34}$$

Inserting the value of C_k from (33) into (31), we get the solution to the problem:

$$U(x,t)=\sum_{k=1}^{\infty}\left\{\frac{\lambda_k^2 L_k\exp(-\alpha\beta_k t)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}+\frac{L_0(\lambda_k)B_0}{\alpha\beta_k L_1(\lambda_k)}\left(1-\exp(-\alpha\beta_k t)\right)\right\}\cdot J_1\left(\lambda_k\sqrt{x}\right). \tag{35}$$

It is obvious that the obtained solution satisfies the boundary condition of problem (11).

Given that

$$L_0(\lambda_k)=\int_0^1 J_1\left(\lambda_k\sqrt{x}\right)dx=\frac{1}{3}\lambda_k F\left[\left\{\frac{3}{2}\right\},\left\{2,\frac{5}{2}\right\},-\frac{\lambda_k^2}{4}\right],$$

$$L_1(\lambda_k)=\int_0^1 J_1^2\left(\lambda_k\sqrt{x}\right)dx=\frac{\lambda_k^2-1}{\lambda_k^2}J_1^2(\lambda_k),$$

$$L_2(\lambda_k)=\int_0^1\left(1-\frac{1}{\lambda_k^2 x}\right)J_1^2\left(\lambda_k\sqrt{x}\right)dx=\frac{1}{\lambda_k^4}\left[\left(\lambda_k^4+1\right)J_1^2(\lambda_k)-\lambda_k^2\right],$$

$$\beta_k=\frac{L_2(\lambda_k)}{L_1(\lambda_k)}=\frac{\left(\lambda_k^4+1\right)J_1^2(\lambda_k)-\lambda_k^2}{\lambda_k^2\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}, \qquad \frac{L_0(\lambda_k)}{L_1(\lambda_k)}=\frac{\lambda_k^3 F\left[\left\{\frac{3}{2}\right\},\left\{2,\frac{5}{2}\right\},-\frac{\lambda_k^2}{4}\right]}{3\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}.$$

Eq. (35) will take the following form:

$$U(x,t)=\sum_{k=1}^{\infty}\left\{\frac{\lambda_k^2 L_k\exp(-\alpha\beta_k t)}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}+\frac{B_0\lambda_k^3 F\left[\left\{\frac{3}{2}\right\},\left\{2,\frac{5}{2}\right\},-\frac{\lambda_k^2}{4}\right]}{3\alpha\beta_k\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}\left(1-\exp(-\alpha\beta_k t)\right)\right\}\cdot J_1\left(\lambda_k\sqrt{x}\right). \tag{36}$$

Using the general solution to the problem, let's derive solutions for private cases. In particular, accepting $U(x,0)=\varphi(x)=A_0$, we have

$$L_k=\int_0^1\phi(x)J_1\left(\lambda_k\sqrt{x}\right)dx=A_0L_0=\frac{1}{3}A_0\lambda_k F\left[\left\{\frac{3}{2}\right\},\left\{2,\frac{5}{2}\right\},-\frac{\lambda_k^2}{4}\right].$$

Therefore,

$$U(x,t)=\sum_{k=1}^{\infty}\left\{\frac{A_0\exp(-\alpha\beta_k t)}{3}+\frac{B_0\lambda_k^3}{3\alpha\beta_k}\left(1-\exp(-\alpha\beta_k t)\right)\right\}\cdot\frac{\lambda_k^3 F\left[\left\{\frac{3}{2}\right\},\left\{2,\frac{5}{2}\right\},-\frac{\lambda_k^2}{4}\right]}{\left(\lambda_k^2-1\right)J_1^2(\lambda_k)}J_1\left(\lambda_k\sqrt{x}\right) \quad (37)$$

These solutions allow us to determine the axial velocities at any point within the cross-section at any given moment in time.

Discussion of experimental results

Eq. (37) determines the velocity change at the current shear point during an unsteady turbulent flow. Let us examine two specific cases.

1. The fluid in the pipe is stationary and is affected by a constant pressure gradient. Acceleration of the fluid occurs, the speed increases from A=0 and becomes equal to the speed of stationary motion. The change in strength at any point in the effective cross-section, depending on time (t), was determined by (37). After computer analysis, flat (Fig.2) and spatial (Fig.3) velocity change graphs were created.

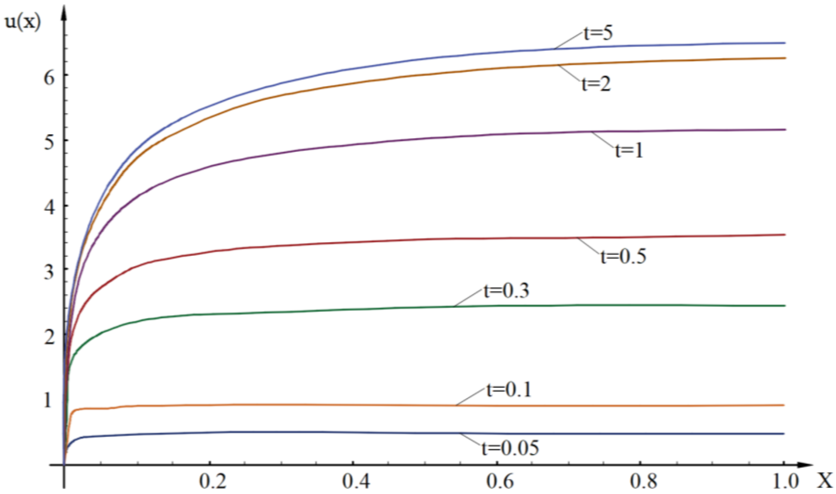

Fig. 2. *Viscous fluid velocity change in case of unsteady plane-parallel flow when A_0=0, B=10* $\alpha = 1$

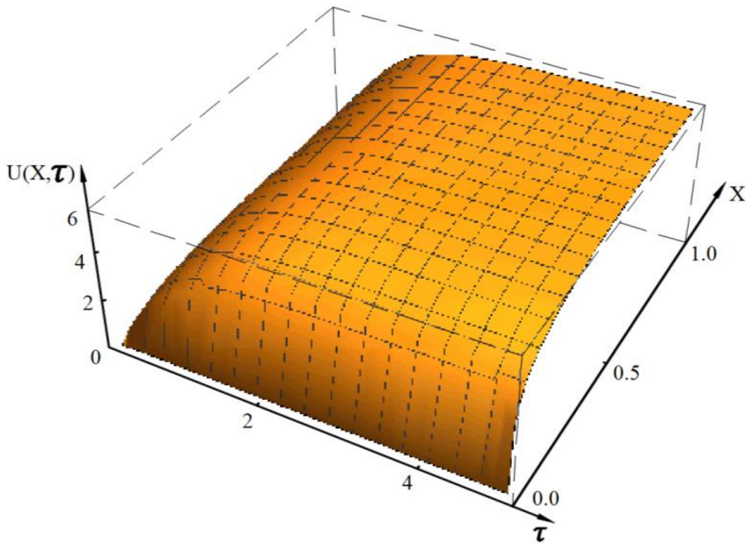

Fig. 3. *Spatial graph of viscous fluid velocity change in case of unsteady plane-parallel flow when A_0 = 0, B = 10* $\alpha = 1$

2. The incoming fluid possesses an initial velocity of A_0=1, and a constant pressure gradient influences the fluid. Velocity variations within the cross-section were derived from (37), and the corresponding graphs, for different time (t), are shown in Figure 4. The spatial image corresponding to these graphs is presented in Figure 5.

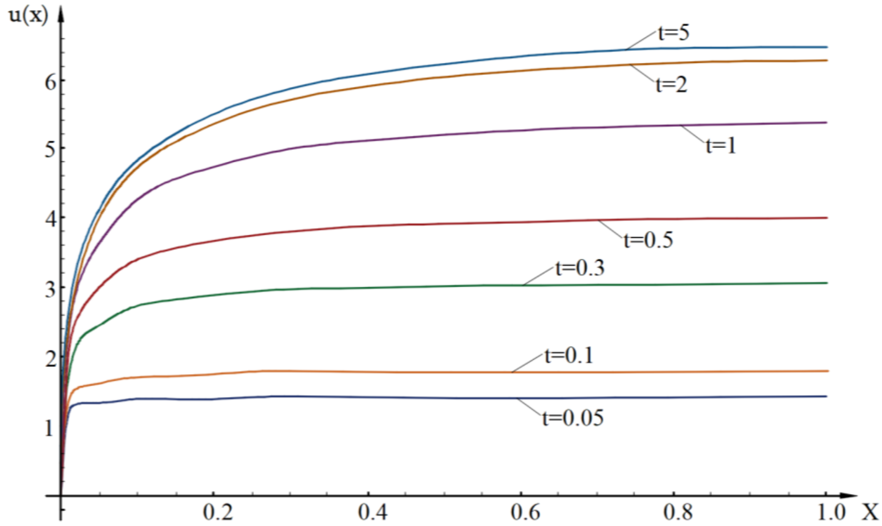

Fig. 4. *Viscous fluid velocity change in case of non-stationary plane-parallel flow when $A_0 = 1$, $B = 5$, $\alpha = 1$*

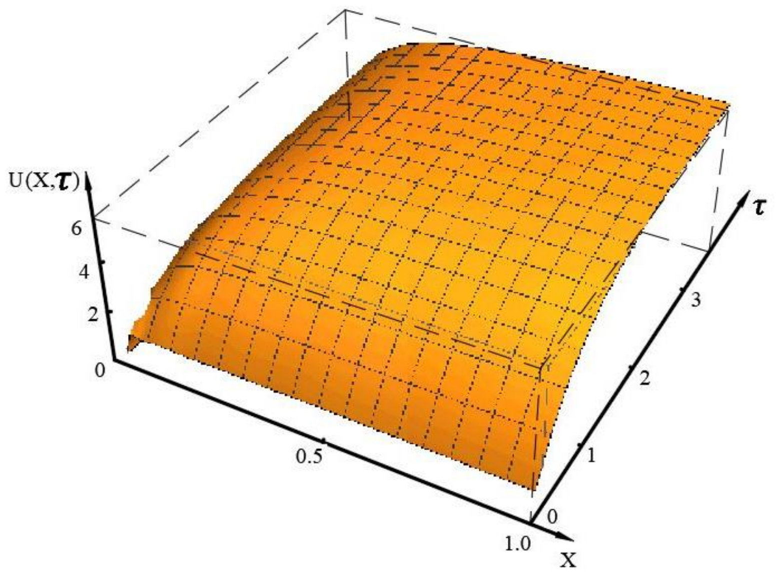

Fig. 5. *Spatial graph of viscous fluid velocity change in case of unsteady plane-parallel flow when $A_0 = 1$, $B = 5$, $\alpha = 1$*

The velocity change graphs show that the fluid in a state of rest begins to move in the layers near the wall forming a boundary layer, in which friction forces arise, and the fluid moves like a solid body outside of it in the core of the flow. Here, there is no sliding between the layers of fluid, hence the presence of frictional forces. Gradually the boundary layer expands and cover the entire cross-section. The dependence of the viscous forces on the distance from the wall extends throughout the entire cross-section of the flow.

Conclusion

1. To study the alterations in hydrodynamic parameters during turbulent unsteady plane-parallel viscous fluid motion, a boundary problem was developed. This problem is constructed on the basis of variations in the dynamic viscosity coefficient of the fluid, considering its dependence on the distance from the stationary wall.
2. An analytical method was developed to solve the boundary value problem, resulting in analytical solutions for velocity variations under unsteady motion conditions. For instance, the hydrodynamic parameters of the flow during plane-parallel motion were observed under conditions of equal velocity distribution and constant pressure gradient in the stationary and inlet sections. The derived patterns of hydrodynamic parameter changes enable conclusions and generalizations to be made.

3. The graphs obtained enable to track down the evolution of hydrodynamic parameters and to assess the quantitative and qualitative impact of the geometric parameters of the problem on these parameters.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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