MECHANICAL BEHAVIOR OF 2D FGP BEAM WITH UNEVEN POROSITY DISTRIBUTION

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Abstract: *This study gives the mechanical behavior of 2D functionally graded porous (FGP) beams using the finite element method. The Matlab code with simple Timoshenko beam elements is written to solve 2D FGP beam problems under distributed load. On the basis of a simple model and approximate results, we can then apply it to analyze this type of structure. The transverse deflections are plotted along the length to provide mechanical views about this structure in reality.*

Keywords*: 2D FGP beam, Uneven porosity, Bending, Transverse deflection.*

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Received: 26.06.2023

Revised: 18.08.2023

Accepted: 19.09.2023

Introduction

In recent years, functionally graded structures have been used in many modern engineering applications due to varying material properties over dimensions which allow them to improve the strength of material, high resistance to temperature shocks and high strength to weight ratio. Authors [1-12] devoted a considerable number of studies to predicting and to understanding the mechanics of functionally graded structures. In [1], the main idea of this paper's developed method was the control over the produced gradient and did not require burning binder phase. The nonlinear bending analysis of FGP beams was investigated using an efficient numerical algorithm associating a meshless collocation technique uses the multiquadric radial basis function approximation method and a higher-order Taylor series-based continuation procedure. Material properties of the FGP beams were described by adopting a modified power-law function taking into account the effect of porosities as in [2]. Besides, the [3] was aimed to develop an efficient and high-performance four-node iso-parametric beam element, which was composed of FGM. In addition, different patterns of material distribution were considered through the height of element. On the other hand, beam's imperfection such as porosity, was taken into account by using the rule of mixture. In order to alleviate the shear locking, MITC was utilized by using tying points. Strain interpolation at some tying points reduced the order of strain functions. The effects of porosity on bending static analysis of FG beams was first introductioned by using a refined mixed finite element beam model. Two different types of porosity namely even and uneven distributions were also considered in [4]. For better understanding of the behaviour of FGM in high temperature environment, a reliable and efficient numerical tool was required for predictions of heat transfer behaviour and thermally-induced stresses in them as [5]. The thermal performance of several engineering devices, such as heat exchangers, volumetric solar receivers and thermal energy storage systems, was improved by open-cell metal or ceramic foams. Among them functionally-graded foams, through which morphological characteristics are variable, look promising. Heat transfer and pressure drop in a functionally-graded foam, with a uniform heat flux entering one of its sides, were investigated numerically in paper [6]. Porosity and cell size variable in the direction of the entering heat flux according to different power-law functions were considered in [6]. A microscale FG Timoshenko beam model was developed for the static bending analysis based on the modified couple stress theory as in [7]. The material properties of the FG microbeams were assumed to vary in the thickness direction and were estimated through the Mori–Tanaka homogenization technique and the classical rule of mixture. The equilibrium equations and the related boundary conditions were derived by using the principal of the minimum total potential energy. In [8], a higher-order element based on the unified and integrated approach of Timoshenko beam theory was developed. A two-node beam element with Hermitian functions of a 5th-degree polynomial (4 dofs per node) was proposed to solve the problems of static and free vibration in this reference and so on. Among many different beam theories, the simple Timoshenko beam model helps us to reduce the computational cost with the resulting error within the allowable range. This article gives the bending behavior of the 2D FGP beam by using a finite element (FE) procedure with simple Timoshenko beam elements respectively.

It is given in four sections. Section 1 shows the introduction as above. Section 2 presents the formulations as well as section 3 shows some essential results. Finally, a few comments are also given in the last section.

Methods

2D FGP beam with uneven porosity distribution

A 2D FGP beam of length *L*, width *b*, thickness *h* and uneven porosity distribution as shown in Fig. 1 is studied in this article. It is made by changing from *c* (ceramic) to *m* (metal) phases through x and z directions.

The volume fraction of the ceramic can be presented in Eq. (1) by following the power law form:

$$V_c(x,z)=\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}. \qquad (1)$$

The material properties can be calculated as below for an uneven porosity distribution with coefficient α and then Fig. 2 also shows the modification of E, respectively:

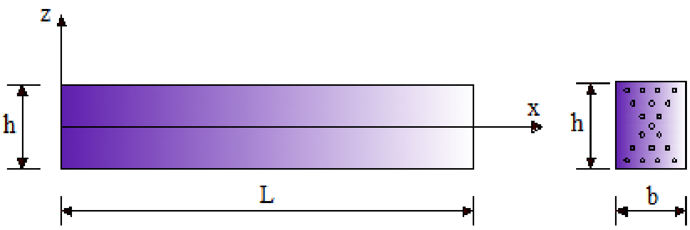

Fig. 1. *A 2D FGP beam with uneven porosity distribution*

$$E(x,z)=(E_c-E_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+E_m-\frac{\alpha}{2}(E_c+E_m)\sin\left(\frac{|z|}{h}\pi\right), \qquad (2)$$

$$G(x,z)=(G_c-G_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+G_m-\frac{\alpha}{2}(G_c+G_m)\sin\left(\frac{|z|}{h}\pi\right), \qquad (3)$$

$$\nu(x,z)=(\nu_c-\nu_m)\left(1-\frac{x}{2L}\right)^{n_x}\left(\frac{1}{2}+\frac{z}{h}\right)^{n_z}+\nu_m-\frac{\alpha}{2}(\nu_c+\nu_m)\sin\left(\frac{|z|}{h}\pi\right). \qquad (4)$$

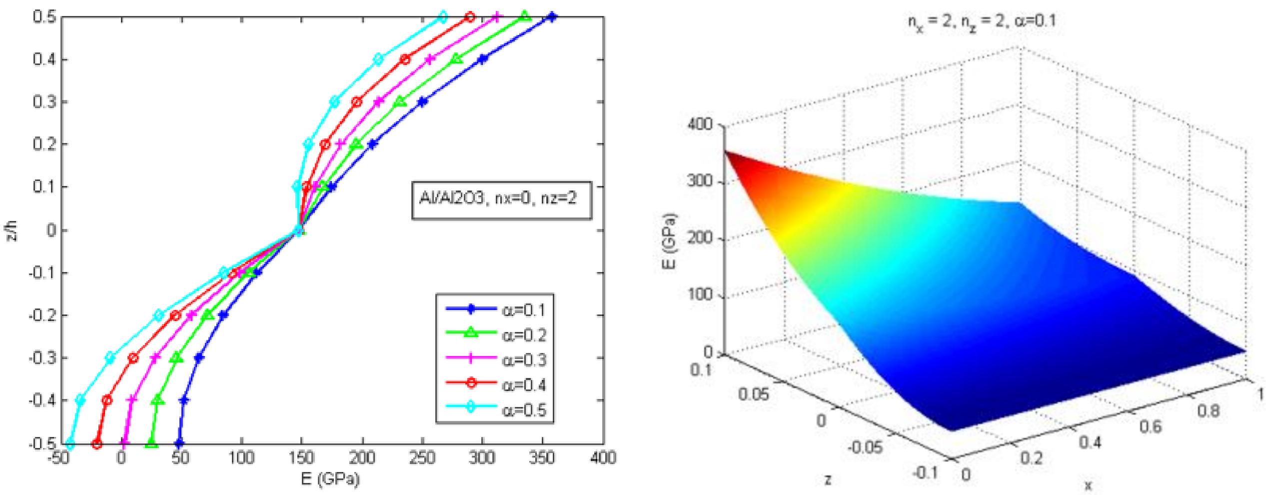

Fig. 2. *The modification of E*

Two dofs associated with a node of a simple Timoshenko beam element are a transverse displacement and a rotation. The beam element stiffness matrix will be derived

$$\mathbf{K}_{el} = \frac{E_{el} I_{el}}{L_{el}^3 (1+\Xi)} \begin{bmatrix} 12 & 6L_{el} & -12 & 6L_{el} \\ 6L_{el} & (4+\Xi)L_{el}^2 & -6L_{el} & (2-\Xi)L_{el}^2 \\ -12 & -6L_e & 12 & -6L_{el} \\ 6L_{el} & (2-\Xi)L_{el}^2 & -6L_{el} & (4+\Xi)L_{el}^2 \end{bmatrix}, \quad (5)$$

with

$$\Xi = \frac{12 E_e I_e}{G_e k A_e L_e^2}, \quad (6)$$

and k =5/6 is called the shear correct factor. According to the principle of minimum total potential energy and after assembly, the transverse deflections can be obtained by solving the following equation:

$$\mathbf{Kd} = \mathbf{F}\,. \quad (7)$$

Results

Some numerical tests are presented to verify the applicability of proposed method. A 2D FGP beam with length L = 1m, b = 0.1m, length to thickness ratio L/h = 20 and distributed load $q = 10^4$ N/m is considered and the material properties can be seen in Table 1 for two-phase.

Table 1. The material properties

Al_2O_3	E_c = 380Gpa	ν_c = 0.3
Al	E_m = 70Gpa	ν_m = 0.3

Table 2. The comparison of normalized maximum tranverse deflection of (CC) 2D FGP beam with α=0

L/h = 20	n_x			
n_z	0.5		2	
	[9]	present	[9]	present
0.5	0.9238	0.9323	1.2279	1.2399
2	1.2414	1.2548	1.5664	1.5838

Firstly, the normalized maximum transverse deflection is formulated by $\bar{v} = 100 E_m b h^3 v_{max} / q / L^4$ as well as all results related to this proposed method for boundary condition (CC_clamped-clamped) are compared with other solutions from [9] by author Karamanli based on Smoothed Particle Hydrodynamics (SSPH) method as shown in Table 2 under coefficient of porosity α = 0. It can be seen that the results obtained from this study are approximate with the solutions of SSPH method. The relative error among these results can be explained by using different approaches.

Finally, to review the influence of material coefficients (n_x, n_z, α) as well as boundary conditions (CC_clamped-clamped, CS_clamped-simply supported, CF_clamped-free) on the deflection of a 2D FGP beam with uneven porosity distribution, Fig. 3 presents curves showing transverse deflections along the length of the beam. It is clear that the deflection value increases when these material parameters increase in all cases.

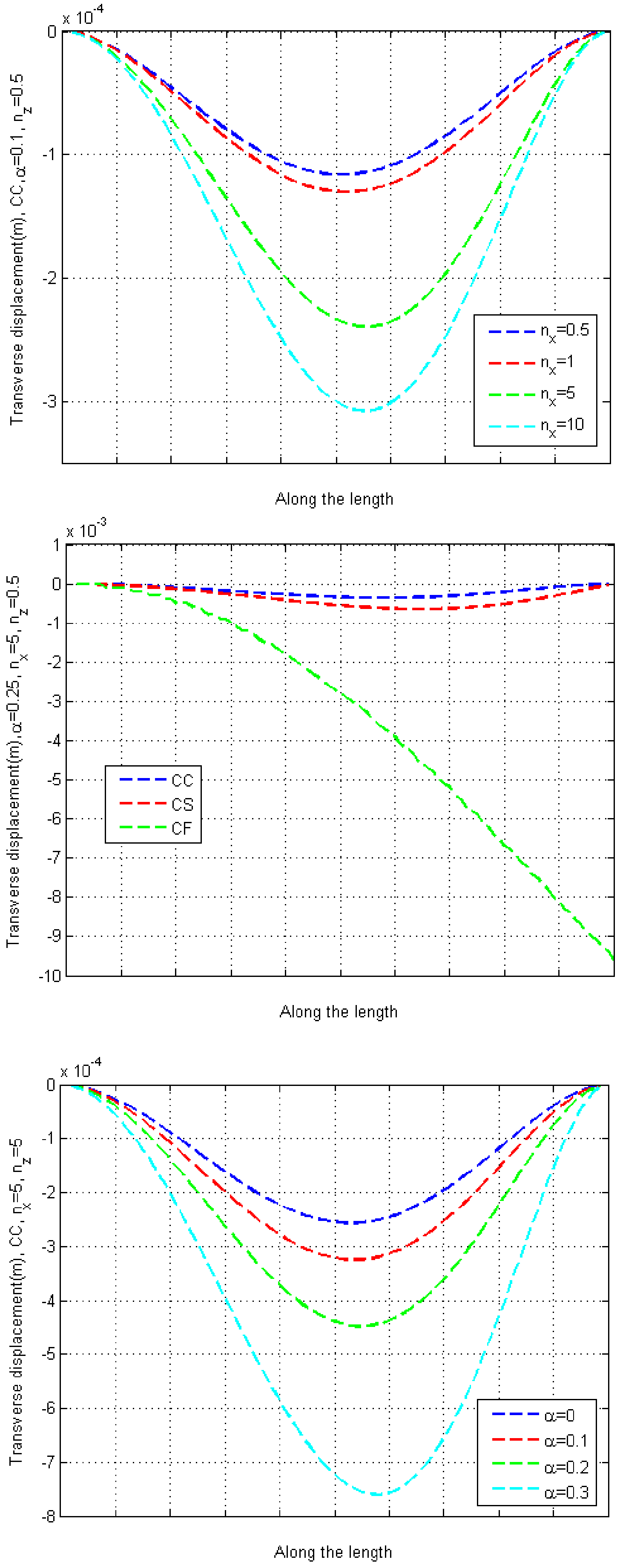

Fig. 3. *The curves of transverse deflection*

Conclusion

In this study, the mechanical behavior of 2D FGP beam with uneven porosity distribution is shown. The simple Timoshenko beam elements based on the FE procedure are applied to analyze the bending behavior of this structure. The results of this article are approximate with other solutions in reference. On the basis of a simple model and approximate results, we can then apply it to analyze this type of structure.

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