AN ENGINEERING COMMENT FOR SIMPLY ACCELERATING SEISMIC RESPONSE HISTORY ANALYSIS OF MID-RISE STEEL-STRUCTURE BUILDINGS



Aram Soroushian¹0*, Abdolreza S. Moghadam¹0, Ahmad Sabzei¹0, Saeed Amiri ²0, Aram Saaed¹0, Ali Yahyapour¹0

¹International Institute of Earthquake Engineering and Seismology, Tehran, Iran ² Polytechnique Montreal, Department of Civil, Geological and Mining Engineering, Montreal, Canada

Abstract: Response history analysis using a time integration method is a powerful versatile tool in accessing structures seismic behaviours. To reduce the analysis run-time, a technique was proposed in 2008 for time integration with steps larger than the steps of ground motions. The technique has been implemented in seismic assessment of frames, buildings, bridges, silos, etc., leading to considerable reductions in the analysis run-time, without notable effect on the response accuracy. The technique has recently been named as the SEB THAAT (Step-Enlargement-Based Time-History-Analysis-Acceleration-Technique). To use the SEB THAAT, the smallest dominant period of the response needs to be available prior to the analysis. In this paper, concentrating on 5-20floor steel-structure buildings, a simple engineering comment is proposed that eliminates this need. As a result, in response history analysis of midrise steel-structure buildings subjected to ground motion, by using the proposed comment, we may reduce the analysis run-time, significantly, without any initial information about the response. The reduction is 50% for the linear analyses.

Keywords: response history analysis, ground motion, mid-rise steelstructure buildings, the SEB THAAT, integration step, excitation step, run-time, accuracy.

Aram Soroushian*

E-mail: a.soroushian@iiees.ac.ir

Received: 11.01.2023 Revised: 30.01.2023 Accepted: 20.02.2023

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Introduction

Response history analysis using a time integration method is a powerful versatile tool, for analysing structures subjected to ground motions, irreplaceable in important analyses [1-3]. After discretization in space, the initial value problem, representing the behaviour of the structural system, is expressible as [4-7]:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{f}_{\text{int}}(t) = \left(\mathbf{M}\Gamma \ddot{u}_{g}(t)\right) \qquad 0 \le t \le \mathbf{t}_{\text{end}}$$
Initial Conditions:
$$\begin{vmatrix} \mathbf{u}(t=0) = \mathbf{u}_{0} \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_{0} \\ \mathbf{f}_{\text{int}}(t=0) = \mathbf{f}_{\text{int}_{0}} \end{vmatrix}$$
 (1)

Additional Constraints: **Q**

In Eq. (1), t and $t_{\rm end}$ imply the time and the analysis time interval; ${\bf M}$ is the mass matrix; ${\bf f}_{\rm int}$ is the vector of the internal force (in linear problems, generally ${\bf f}_{\rm int} = {\bf Ku} + {\bf Cu}$, where ${\bf K}$ and ${\bf C}$ stand for the matrices of stiffness and viscous damping, respectively); $\ddot{u}_{\rm g}(t)$ implies the single-component ground acceleration, and Γ is a vector with the size of the degrees of freedom, needed for matrix multiplication and considering spatial changes of $\ddot{u}_{\rm g}(t)$; ${\bf u}(t)$; $\dot{\bf u}(t)$, and $\ddot{\bf u}(t)$, denote the vectors of displacement, velocity, and acceleration, relative to the ground; ${\bf u}_0$, $\dot{\bf u}_0$, and ${\bf f}_{\rm int}_0$, define the initial status (generally all zero), and ${\bf Q}$ implies the limiting conditions due to nonlinearity. (When the ground motion is multi-component, Γ and $\ddot{u}_{\rm g}(t)$ will change to a matrix and a vector, respectively). The core of response history analysis is an approximate step-by-step computation, widely known as direct time integration (see Fig. 1 and [1,3,6]). The resulting

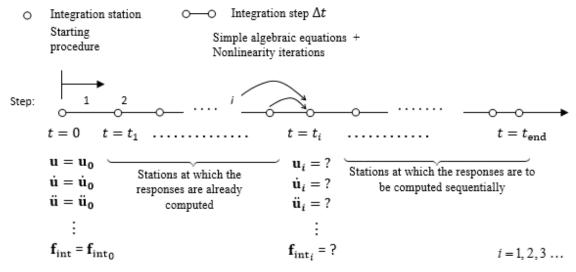


Fig. 1. A pictorial review of direct time integration analysis of Eq. (1) [3]

responses are inexact, and the analysis run-times are generally considerable [3,6]. The integration step is the analysis parameter, with adverse effects on the run-time and the accuracy [3]. Accordingly, a regulation for assigning an appropriate value to Δt is essential. A simple comment, broadly accepted in practice, is as follows [2,3,8]:

$$\Delta t \cong \leq \operatorname{Min}\left(\frac{T}{X}, h, {}_{f}\Delta t\right)$$
 (2)

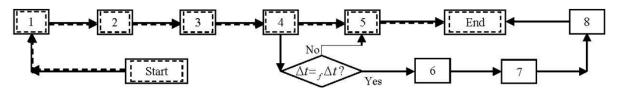
In Eq. (2), T is the smallest dominant period in the time history of the response, h implies the largest integration step preserving numerical stability and consistency, $f\Delta t$ denotes the step, by which, the ground motion is digitized, and X equals 10 for linear systems, 100 for nonlinear systems not involved in impact, and 1000 for systems involved in impact. When, the dominant term in the right hand side of Eq. (2) is $_f\Delta t$, the difference between $f\Delta t$ and the next smallest term in the right hand side of Eq. (2) implies the effort required mainly to account for all the excitation data. In order to eliminate or lessen this effort, in 2008, a technique was proposed [3,9,10], that replaces the earthquake record with a record digitized in larger steps. The technique, which is recently named "the SEB THAAT (Step-Enlargement-Based Time-History-Analysis-Acceleration-Technique)" [10], is formulated such that to prevent any negative effect because of the record replacement on the convergence of the computed response to the exact response. Considering that convergence is the main essentiality of approximate computations [11,12], the expectation from the SEB THAAT is to reduce the computational effort with negligible effect on the response accuracy. In view of the studies carried out since 2008, the SEB THAAT has been successful, in earthquake engineering and beyond; see Table 1 [3,10] and [13]. Nevertheless, to set the scaling value of the record's step (i.e. how much to enlarge the step), some information about the response is needed, prior to the analysis [3,10]. The objective in this paper is to eliminate this need for a special class of analyses. In view of the number and social importance of buildings with 5-20 floors, hereafter referred to as mid-rise buildings, seismic response history analysis of steel-structure mid-rise buildings is considered as the special class of the analyses. Accordingly, by achieving the objective, the computational effort needed for a large number of response history analyses will be simply reduced. The simplicity in increasing the efficiency and the amount of the increase can encourage engineers to use response history analysis in practice, as well. For achieving the objective, the main attention is paid to Eq. (2), the current design and analysis practice, and the trends of the future advancements. A brief review on the SEB THAAT is presented first. An engineering comment for selecting the enlargement scale is introduced next. The effectiveness of the comment is tested afterwards, and finally, the paper is concluded with a discussion on practical issues, as well as an overview of the achievements.

Table 1. Some past tests on the SEB THAAT with regard to seismic response history analysis

System	Reduction in run-time (%)
A thirty-storey building	50
3-component earthquakes	66
Silo	77
Water tanks and Silos	66
Bridges	45-80
Residential buildings	50-87
An earth dam	< 80
Milad telecommunication tower	50-70
Different lifelines	50-90

A brief look at the SEB THAAT

An overview of the SEB THAAT, its formulation, limitations, challenges, and future prospects, is recently presented in [10]. Therefore, for brevity, only application of the SEB THAAT to direct time integration analysis is referred to in Fig. 2, and the most important features are reviewed as follows:



- 1. Define the structural model and the excitation (the excitation digitization step equals $_f\Delta t$)
- 2. Select the integration method
- 3. Select the details of the nonlinear solution (if needed)
- 4. Select the integration step Δt , regardless of the SEB THAAT
- 5. Step-by-step direct time integration using Δt as the integration step
- 6. Assign a value to the enlargement scale n
- 7. Use the SEB THAAT to change the excitation to an excitation digitized at step $n_f \Delta t$
- 8. Step-by-step direct time integration using $n_f \Delta t$ as the integration step

Fig. 2. Application of the SEB THAAT to arbitrary direct time integration analysis

- 1. The SEB THAAT has a mathematical basis, with the aim and formulation to preserve the responses' convergence, without necessarily avoiding changes in characteristics of the earthquake record (see [9]).
- 2. Considering Eq. (2) as a basis for selection of the integration step, the SEB THAAT is to be applied, only when $_f\Delta t$ dominates the right hand side of Eq. (2), i.e.

$$_f \Delta t \le \operatorname{Min}\left(\frac{T}{X}, h\right)$$
 (3)

_ _ Direct time integration

Direct time integration after applying the SEB THAAT

It should however be noted that Eq. (2) is not rigorous [2,3,8,14], and hence, the SEB THAAT may be successful, even when Eq. (3) is invalid; see for instance [15].

- 3. The formulation of the SEB THAAT enables consideration of arbitrary real number greater than one as the step enlargement scale [16].
- 4. With appropriate details of nonlinear solution and sufficient computational facility, the SEB THAAT can be as successful in application to nonlinear analyses, as it is in application to linear analyses; see [17].
- 5. While in linear analyses application of the SEB THAAT causes reduction in the analysis run-time, *R*, obtainable from

$$R = 100 \frac{n-1}{n} \% {4}$$

(n is the step enlargement scale), in nonlinear analyses, the reductions are not necessarily obtainable from Eq. (4) [3].

- 6. With a basis on convergence of the computed results, the SEB THAAT may be successfully applicable in fields different from structural dynamics and earthquake engineering [13].
- 7. Compared to the SEB THAAT, direct down sampling [18] is less effective, i.e. the accuracy of the target response obtained from the analysis when using the SEB THAAT is more than when using direct down sampling instead of the SEB THAAT [19].

It is also worth noting that, the future of the SEB THAAT is promising, due to its simplicity, remarkable effectiveness, and everyday more availability of different data as digitized records with smaller digitization steps.

A new engineering comment

For the SEB THAAT to reduce the computational effort, a positive value larger than one should be assigned to the step enlargement scale, n; see also [3,10]. Besides, the greater the n, the more will be the reduction in the run-time, especially for linear analyses [3,9,10]; see Eq. (4). Accordingly and in view of Eq. (2), for the analysis most efficiency and good accuracy of the response, the excitation step $n_f \Delta t$ should govern the right hand side of Eq. (2). Accordingly, when $h \to \infty$ (broadly recommended [3,4,6,20,21]):

$$n \cong \frac{T}{X_f \Delta t},\tag{5}$$

which is effective, when the resulting n is greater than one. (The reason of using an approximation sign in Eq. (5) is that, different from h and $f\Delta t$, the T/X in Eq. (2), the definition of T, the values of X, and the form of Eq. (2), are not rigorous (see [3,14])).

The existing seismological instrumentations provide the capability of recording ground motions, in steps, as small as 0.004 sec [22]. These instrumentations are in continuous progress, towards smaller digitization steps; see [23]. Considering this, along with the about largest digitization steps currently in use [24], we can conclude that:

$$_f \Delta t \le 0.02 \,\mathrm{sec}$$
 , (6)

and,

$$\frac{T}{X_f \Delta t} \ge \frac{50}{X} T \,. \tag{7}$$

Therefore, considering that smaller values of n will be more reliable from the standpoint of response accuracy, it is practically acceptable to replace Eq. (5), with

$$n \cong \frac{50 \, T}{X} \,. \tag{8}$$

Accordingly, if for a class of structural analyses there exists a minimum for T (see Eq. (2)), i.e. T_{\min} , such that:

$$n \cong \frac{50 \, T_{\min}}{Y} > 1 \,, \tag{9}$$

the n is suitable in application of the SEB THAAT in that class without any information about the response.

Taking into account 5-20-floor steel-structure buildings, designed according to the Iranian codes^{1,2}, and in agreement with the International code³ and [25,26], the least dominant periods of the displacements linear oscillations, because of ground motions, satisfy (displacements, velocities, and accelerations, are the unknowns generally being computed first in time integration analysis):

$$T_{\min} \cong \ge 0.38 - 0.40 \,\text{sec}$$
 (10)

¹ BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

² INBR (Iranian National Building Regulations), Iranian National Building Code, Part 10-Steel Structures, Iran, 1993 (in Persian).

³ ICC (International Code Council), IBC - International Building Code, Club Hills, USA, 2003.

Besides, nowadays the technology of buildings construction proceeds towards lighter designs, and generally less stiff structural systems and larger values of T_{\min} [27]. Considering these, in application of the SEB THAAT to analysis of 5-20-floor steel-structure buildings, the following comment is reasonable:

$$n=2. (11)$$

It is worth noting that Eq. (11) is in agreement with the experiences on the SEB THAAT's application, reviewed in [10]; see also Table 1. Equation (11) implies an engineering comment, for implementation of the SEB THAAT [9] which, is considered above for linear analyses, and simply reduces the analysis run-time for about 50%. Nonlinearity because of inelastic behaviour is inherent to structures subjected to severe earthquakes [1], and generally causes increase of both T_{\min} and X. Besides, the T/X in Eq. (2), the definition of T, the form of Eq. (2), and especially the values of X are not rigorous [14]. Furthermore, seismic standards mostly permit linear analyses for ordinary building structures^{4,5,6,7,8,9,10}, and the only seismic standard, with a procedure for nonlinear response history analysis, i.e. NZS 1170.5:2004¹¹, also proposes a regulation to check the accuracy of the response. Accordingly, it is reasonable to use Eq. (11) in application of the SEB THAAT to analysis of 5-20-floor steel-structure buildings inelastic behaviours, as well. Nevertheless, because of the iterative nonlinear solutions [7,28,29], using Eq. (11) will not lead to twice faster analysis when the behaviour is inelastic. Consequently, we can use Eq. (11) in application of the SEB THAAT to seismic response history analysis of 5-20-floor steel-structure buildings, and reduce the analysis run-time, without prior information about the response. This claim is tested in the next section, considering quiescent condition at t = 0 and S.I. system of units for all computations.

Numerical study

Simple test on a six-floor steel shear frame

Consider response history analysis of the system defined in Fig. 3 (g stands for the acceleration of gravity) and Table 2, by the C-H method [30] ($\rho_{\infty}=0.8$). The top displacement is set as the target response. The suitability of using Eq. (11) is tested via analysis with the integration step $\Delta t = {}_f \Delta t$ and a second analysis using $\Delta t = 2{}_f \Delta t$. The sufficient accuracy of the response and the validity of Eqs. (3) and (10) are displayed in Figs. 3 and 4, while the numbers of integration steps (37500 and 18750, for Figs. 4(a) and 4(b), respectively) imply 50% reduction in the analysis run-time.

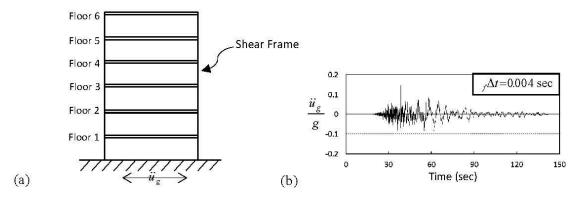


Fig. 3. Pictorial introduction to the first example: (a) Structural model, (b) Ground motion

¹⁰ NRCC (National Research Council Canada), National Building Code of Canada, Canada, 2005.

⁴ NZS (New Zealand Standards), NZS 1170.5: 2004 Structural Design Actions, Part 5: Earthquake actions, New Zealand, 2004.

⁵ BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

⁶ INBR (Iranian National Building Regulations), Iranian National Building Code, Part 10-Steel Structures, Iran, 1993 (in Persian).

⁷ ICC (International Code Council), IBC - International Building Code, Club Hills, USA, 2003.

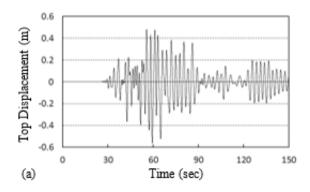
⁸ BCJ (Building Centre of Japan), Structural Provisions for Building Structures, Tokyo, Japan, 2001.

⁹ EAK, Greek Code for Seismic Resistant Structures, Athens, Greece, 2000.

¹¹ NZS (New Zealand Standards), NZS 1170.5: 2004 Structural Design Actions, Part 5: Earthquake actions, New Zealand, 2004.

 Table 2. Main properties of the structural model in the first example

Droporty	Floor (i) / Mode (i)										
Property	1	2	3	4	5	6					
Mass $(m_i) \times 10^{-9}$	1.8	1.8	1.8	1.8	0.6	0.6					
Stiffness $(k_i) \times 10^{-11}$	1.20	0 1.20	1.20	1.20	0.20	0.20					
Natural period (T_i)	2.7191	1.4919	0.7695	0.6544	0.4973	0.4088					
Damping	Classical dan	Classical damping [2], considering 2% damping for the 1 st and 3 rd natural modes									



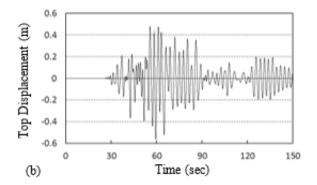


Fig. 4. Target response computed for the first example by the C-H time integration method ($\rho_{\infty}=0.8$) using an integration step equal to: (a) $_f\Delta t$, (b) $_f\Delta t$ (by means of the SEB THAAT)

A fifteen-floor steel-structure building

Consider the 15-floor steel-structure building displayed in Fig. 5. There are two identical axes of symmetry in the plan; the lengths of the spans are four meters; the bracings are placed at the last spans of the surrounding frames; the structural system is dual¹², and the other basic details are reviewed in Table 3. The structure is designed in a previous study [31], based on the Iranian standards^{13,14}, for the lowest total cost of construction using a fully constrained optimal criterion [31,32]. The members' cross-sections are as reported in Table 4.

Four two-component records, of the historically most devastating earthquakes in Iran, are considered, as the ground motion records (see Fig. 6) (for the first earthquake, the two components are identical). The structure is modelled as a three dimensional shear frame [2,24]. The translational natural periods are in the interval (0.035 sec 0.8 sec). The seismic response history analyses are carried out using the average acceleration time integration method [33], twice, for each two-component record; once, ordinarily, and once, after application of the SEB THAAT [9,10], considering Eq. (11). The acceleration at top, the mid-height displacement, and the base shear, are considered as the target responses. The computed histories are depicted in Figs. 7-10, where, for further clarity, the mid-sections of the time histories are not displayed in Figs. 8-10. These figures show that Eq. (11) may present an adequate comment for assigning a value to n in application of the SEB THAAT in response history analysis of mid-rise steel-structure buildings.

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¹² BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

¹³ Ibid.

¹⁴ INBR (Iranian National Building Regulations), Iranian National Building Code, Part 10-Steel Structures, Iran, 1993 (in Persian).

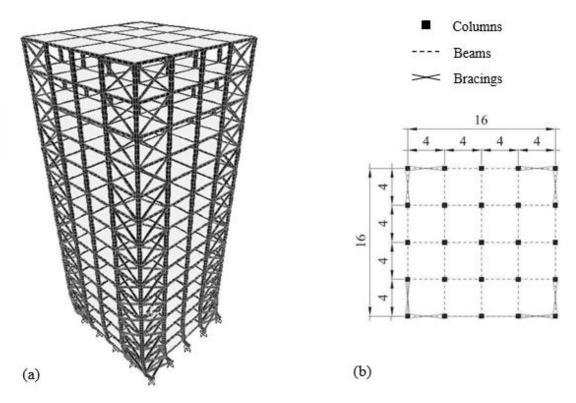


Fig. 5. The building structure in the second example: (a) Schematic view, (b) Top view

Table 3. Basic details for the structural systems in the second and third examples

Material	Steel (ST-37)
Occupancy	Residential (in Shiraz, Iran; mass: 175000 and 200000 Kg for the roof and other floors respectively)
Seismic zone factor	0.35 ¹⁵
Soil type	II $(375 \text{ m/s} \le \overline{Vs} \le 750 \text{ m/s})^{16 \text{ a}}$
Floor height	3 meters
Bracing	X
Damping	Negligible (considered zero in the analyses)

^a \overline{V}_S is the velocity of shear waves.

Table 4. Members' cross-sections for the structural system introduced in Fig. 5 and Table 3

Floor	Inner	Peripheral	Corner	Inner beams	Peripheral beams	Bracings
(from	columns	columns	columns			
ground)						
1	IPB400 a	Box400*12.5	Box360*10	IPE 300	2IPE160	2L130*12
2-3	IPB400 a	Box400*12.5	Box360*10	PL200*10 + 2PL240*20	2IPE200	2L130*12
4-5	Box320*10	Box400*16	Box360*10	2IPB200	2IPE200	L180*16
6-7	Box320*10	Box360*16	Box320*10	PL200*10 + 2PL240*20	2IPE200	2L110*10
8-11	Box320*10	Box360*12.5	Box320*10	PL200*10 + 2PL240*20	INP240	2L110*10
12-15	Box320*12.5	Box320*10	Box280*8	PL150*10 + 2PL180*15	PL220*6 + 2PL120*10	L130*12

^a For construction problems, the properties in the weaker direction of the cross-section are considered as the properties in both directions.

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¹⁵ BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

¹⁶ Ibid.

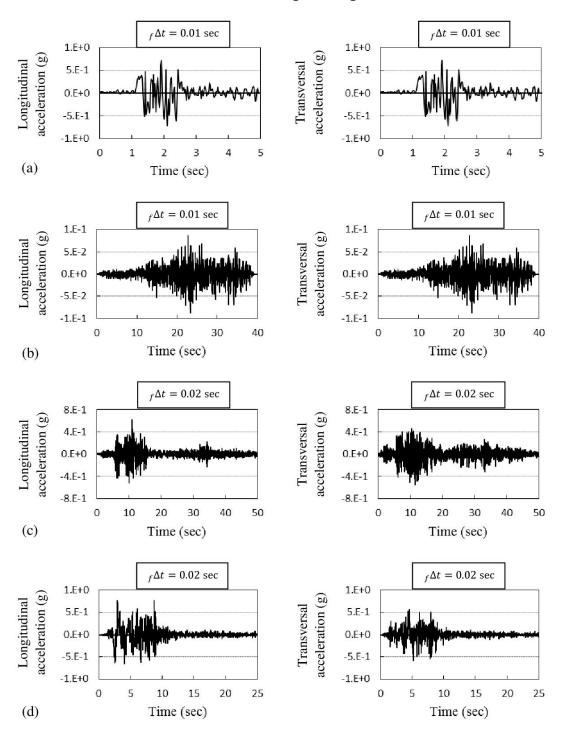


Fig. 6. Two-component records of the four most devastating earthquakes in Iran, as the ground motion records in the second example: (a) Naghan (1977), (b) Tabas (1978), (c) Abbar (1990), (d) Bam (2003)

Sixty-five 10-20-floor steel-structure buildings

In this section, the adequacy of Eq. (11) is studied, in view of 25 ten-, 25 fifteen-, and 15 twenty-story steel-structure buildings, each with two identical axes of symmetry, X bracings with the configuration displayed in Fig. 11, and span lengths constant throughout the structure, equal to either of the followings (see also [31]):

$$s = 4, 5, 6 \text{ (meters)}.$$
 (12)

Two two-component ground motions, selected, based on the soil type and shear wave speed, are applied, at the ground level, in the principle directions of the structures (see also Fig. 12 and Tables 3, 5, 6). The response

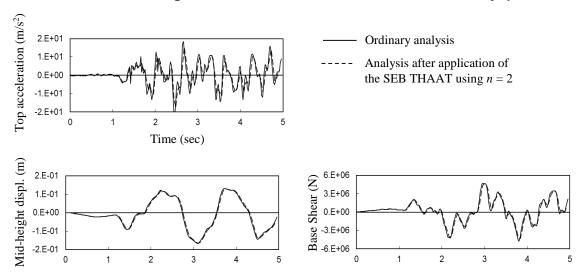


Fig. 7. Target responses computed for the second example when subjected to the records in Fig. 6(a)

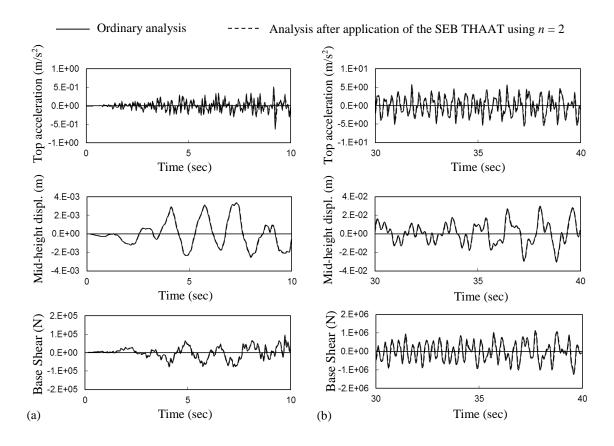


Fig. 8. Target responses computed for the second example when subjected to the records in Fig. 6(b): (a) The starting ten seconds, (b) The ending ten seconds

history analyses are carried out using the average acceleration method [33], once with the step $\Delta t = {}_f \Delta t$, and then with the step $\Delta t = 2{}_f \Delta t$ (after applying the SEB THAAT considering Eq. (11)). The maximum relative difference between the two responses in the L_{∞} norm [34] is reported in Tables 7-9. Smallness of the reported values, that the ordinarily computed responses are not exact [2-4,6,35], and the 50% reduction in the analysis run-time, imply the good performance of the SEB THAAT when using Eq. (11).

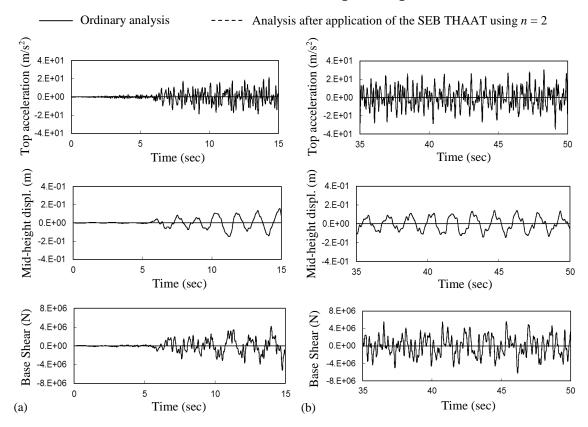


Fig. 9. Target responses computed for the second example when subjected to the records in Fig. 6(c): (a) The starting fifteen seconds, (b) The ending fifteen seconds

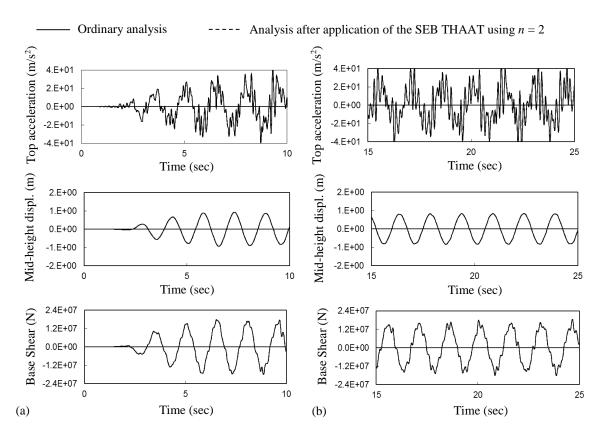


Fig. 10. Target responses computed for the second example when subjected to the records in Fig. 6(d): (a) The starting ten seconds, (b) The ending ten seconds

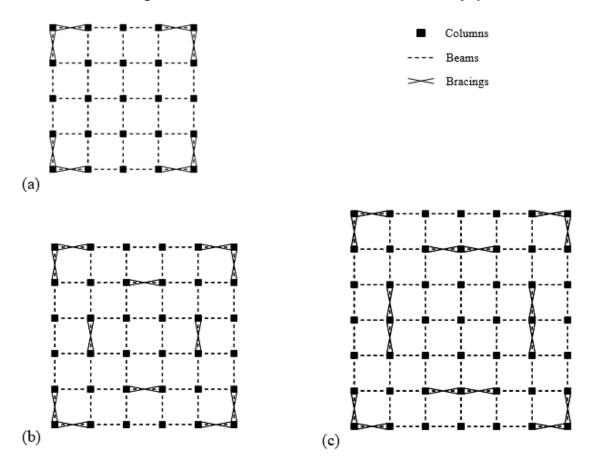


Fig. 11. Structural plans in the third example: (a) Plans with four spans, (b) Plans with five spans, (c) Plans with six spans

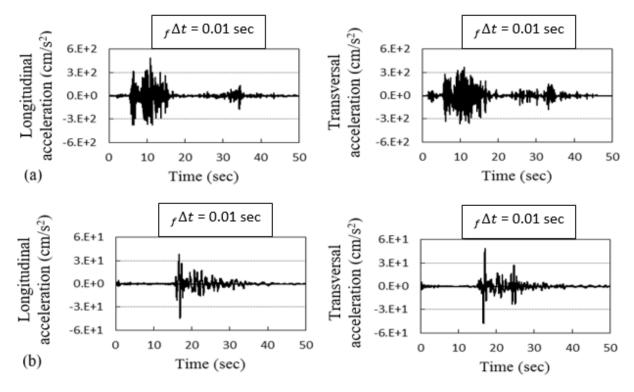


Fig. 12. Records of the ground motions in the third example: (a) Manjil (1990), (b) Koujoor (2004)

Table 5. Systems of the buildings structures in the third example according to the Iranian codes 17,18 a

System	10-floors	15-floors	20-floors
Dual consisted of special moment frames and concentrically braced frames	+	+	+
Special moment frames	+	+	+
Dual consisted of intermediate moment frames and concentrically braced frames	+	+	+
Intermediate moment frames	+	+	-
Ordinary concentrically braced frames	+	+	-

a "+" and "-" imply being and not being considered in the design code (and in this paper), respectively

Table 6. Groups of identical structural members in the sixty-five buildings in the third example

Buildings	Floors with identical structural members a, b
10-floor buildings	1-2-(3, 4)-(5, 6)-(7, 8, 9, 10)
15-floor buildings	1-(2, 3)-(4, 5)-(6, 7)-(8, 9, 10, 11)-(12, 13, 14, 15)
20-floor buildings	1-2-(3, 4)-(5, 6)-(7, 8)-(9, 10, 11, 12)-(13, 14, 15, 16)-(17, 18, 19, 20)

a The numbers in each "()" address the floors' numbers with identical structural members (groups in height)

Table 7. Differences in the L_{∞} norm, between the responses obtained when using the SEB THAAT and Eq. (11) and the responses computed using $\Delta t = {}_f \Delta t$, for the 10-floor buildings in the third example (%)

Structural system	Top acceleration	Mid-height displacement	Base shear
Dual consisted of special moment frames and concentrically braced frames	≤5.39	≤ 0.83	≤ 1.79
Special moment frames	≤ 5.92	≤ 0.89	≤ 0.72
Dual consisted of intermediate moment frames and concentrically braced frames	≤ 5.46	≤ 0.66	≤ 1.91
Intermediate moment frames	≤ 5.77	≤ 0.83	≤ 1.17
Ordinary concentrically braced frames	≤ 6.20	≤ 0.89	≤ 2.55

Table 8. Differences in the L_{∞} norm, between the responses obtained when using the SEB THAAT and Eq. (11) and the responses computed using $\Delta t = {}_f \Delta t$, for the 15-floor buildings in the third example (%)

Structural system	Top acceleration	Mid-height displacement	Base shear
Dual consisted of special moment frames and concentrically braced frames	≤0.53	≤ 1.67	≤ 5.98
Special moment frames	≤ 0.15	≤ 0.57	≤ 5.43
Dual consisted of intermediate moment frames and concentrically braced frames	≤ 0.60	≤ 1.80	≤ 5.96
Intermediate moment frames	≤ 0.13	≤ 0.59	≤ 5.90
Ordinary concentrically braced frames	≤ 0.62	≤ 1.46	≤ 5.58

Table 9. Differences in the L_{∞} norm, between the responses obtained when using the SEB THAAT and Eq. (11) and the responses computed using $\Delta t = {}_f \Delta t$, for the 20-floor buildings in the third example (%)

Structural system	Top acceleration	Mid-height displacement	Base shear
Dual consisted of special moment frames and concentrically braced frames	≤0.23	≤ 0.79	≤ 5.38
Special moment frames	≤ 0.14	≤ 0.98	≤ 5.63
Dual consisted of intermediate moment frames and concentrically braced frames	≤ 0.28	≤ 1.15	≤ 5.13

¹⁷ BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

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b The seven groups with identical structural members in plan are: internal columns, side columns, corner columns, internal beams, peripheral beams, internal bracings (only for cases in Figs. 11(b) and 11(c)) and side bracings

¹⁸ INBR (Iranian National Building Regulations), Iranian National Building Code, Part 10-Steel Structures, Iran, 1993 (in Persian).

A 15-floor steel-structure building with nonlinear behaviour and non-classical damping

A 15-floor steel-structure building model is under study for the top displacement and base shear; see Fig. 13 and Table 10. The behaviour is nonlinear, with attention to Fig. 14, with regard to which, for the top displacement, $T \cong 1.0 > 0.40$ sec; see also Eq. (10). This is also in agreement with the fact that the first seven natural periods of the system are greater than 0.4 sec (see the mode shape in Fig. 15), nonlinear behaviour can

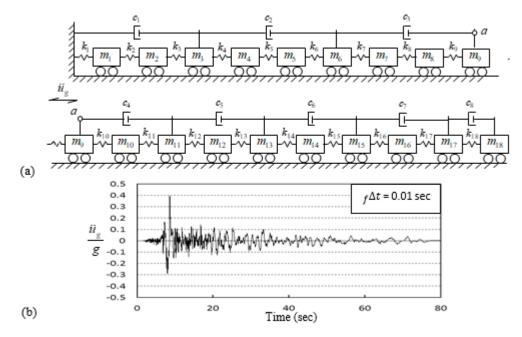


Fig. 13. Pictorial introduction to the fourth example: (a) Structural model, (b) Ground motion

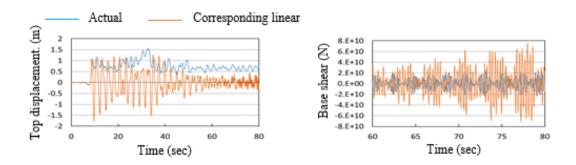


Fig. 14. Exact responses of the fourth example

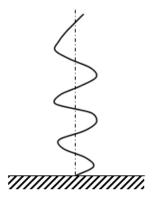


Fig. 15. Shape of the seventh natural mode ($T_7 = 0.42 \text{ sec}$), for the linear system corresponding to the nonlinear system introduced in Fig. 13(a) and Table 10

Table 10. Main properties of the structural model in the fourth example

D.										i								
Property	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$10^{-9} \times m_i$	3	3	3	3	3	3	3	3	1.5	1.5	1.5	1.5	1.5	1.5	0.5	0.5	0.5	0.5
$10^{-12} \times k_i$	2	2	2	2	1.2	1.2	1.2	1.2	0.6	0.6	0.6	0.6	0.6	0.6	0.1	0.1	0.1	0.1
$10^{-8} \times c_i$	12	8	6	2.5	2.5	1.5	0.5	0.2										
$10^2 \times u_{y_i}$	8	8	8	8	8	8	5	5	5	5	5	5	5	5	3	3	3	3

be considered an extension of linear behaviour, the first few natural modes play the main role in the structural behaviour, and that the linear response is a combination of the responses in different natural modes. (In view of Fig. 14, for the base shear, $T \cong 0.1$ sec.) Consequently, we can expect the good performance of the SEB THAAT and Eq. (11) when applied to the response history analysis of the structure, especially for the top displacement. It is meanwhile worth noting that, different from the previous examples, where the damping was classical viscous (in the first example) and zero (in the second and third examples), in view of Fig. 13(a) and Table 10, the damping is non-zero and non-classical viscous in this example [2,36,37].

The model is analysed twice using the average acceleration method [33], and twice using the Wilson- θ [38,39] ($\theta = 1.4$) method. In the first analyses by either method, the integration step is set to $_f\Delta t$ (obtained from Eq. (2) for the first target response). The analyses are then repeated using $\Delta t = 2 f \Delta t$, after implementation of the SEB THAAT. The fractional time stepping method [40,41] is used for nonlinearity solution, and the nonlinearity tolerance and the maximum number of iterations are considered equal to 1E-6 and 5, respectively (see [17,40,41]). The results are displayed in Fig. 16, and are evaluated, taking into account that the responses of nonlinear dynamic analyses may be inaccurate even significantly regardless of the SEB THAAT [1,3,7,28,42-46]. For the first target response, i.e. the top displacement, the responses obtained with or without application of the SEB THAAT coincide, when analysing with either integration method. For the base shear, the two responses are close, in analysis with the average acceleration method [33]. In analysis with the Wilson- θ [38,39] ($\theta = 1.4$) method, however, the computed two base shears are evidently different. To better study the difference between the two base shears, obtained from the Wilson- θ [38,39] (θ = 1.4) method (see Fig. 16(b)), the results of analysis with very small steps is displayed in Fig.16(c). In view of this figure, both of the base shears computed by the Wilson- θ [38,39] ($\theta = 1.4$) method (when applying and not applying the SEB THAAT) differ significantly from the exact base shear (displayed in Fig. 16(c)). In more detail, the two differences (between the two base shears in Fig. 16(b) and the exact base shear in Fig. 16(c)) are much larger than the difference between the two base shears in Fig. 16(b). Therefore, the accuracy of the base shears in Fig. 16(b), is acceptable, in the sense that replacing the base shear obtained using $\Delta t = {}_f \Delta t$ with the base shear obtained using $\Delta t = 2 {}_f \Delta t$ (both displayed in Fig. 16(b)) does not imply meaningful change in the response accuracy. Even more, returning to the origin of the Wilson- θ method [38], one of the main purposes of the Wilson- θ method is to filter high mode responses out of the response [47]. This filtering, which is broadly known as numerical damping of time integration methods [3,4,6,20,21,48,49], is essential, when we are seeking the responses of the structural model before the discretization resulting in Eq. (1). The discretization, replaces the structural model (with infinite number of degrees of freedom) with the mathematical model in Eq. (1) (with finite number of degrees of freedom), in the price of spurious high frequency oscillations in the response, which can be eliminated by numerical damping [3,6,49]. (Numerical damping can also eliminate real high modes with small contribution in the response, but computed erroneously because of largeness of the $\Delta t/T$ [3,6,47-49]). Considering this, when using the Wilson- θ method [38], the purpose of the analysis may be different from achieving good accuracy compared to the exact response reported in Fig. 16(c). As a result, because of "selection" of the integration method, the base shears in Figs. 16(a) and 16(b) not only show the good performance of SEB THAAT when using Eq. (11), but can also be considered sufficiently accurate.

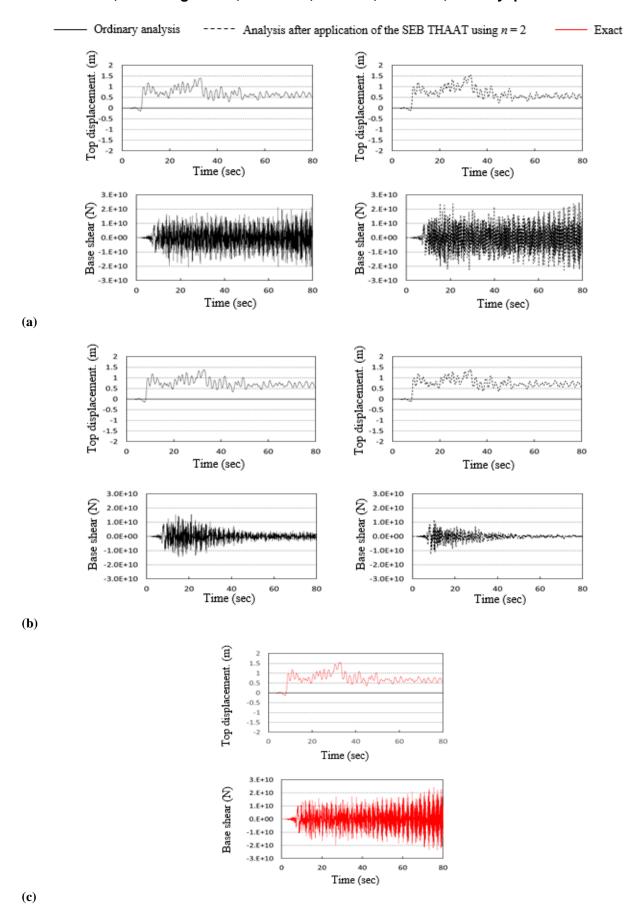


Fig. 16. Target responses of the fourth example: (a) Computed using the average acceleration method, (b) Computed using the Wilson- θ ($\theta = 1.4$) method, (c) Exact

For further clarity, the following two questions are to be answered, as well:

- (a) What is the reason of the different performances of the SEB THAAT using Eq. (11), in Figs. 16(a) and 16(b)? Meanwhile, why are the base shears in Figs. 16(a) (and Fig. 16(c)) and 16(b) such different that while in Fig. 16(a) (and Fig. 16(c)) the base shear increases with time, in Fig. 16(b), the base shear decreases with time?
- (b) What is the reason of the negligible difference between the top displacements in Fig 16(b), while the difference between the two base shears in Fig. 16(b) is recognizable?

The answer to Question (a) lies in the fact that Eq. (2) is not rigorous; besides other ambiguities, it is not clear why the integration method does not affect the selection of the integration step. More specifically, the numerical damping referred to in the previous discussion is very different in the average acceleration and Wilson- θ methods, with no influence on Eq. (2). The difference is evident in Fig. 17, for problems with classical viscous damping [50], and is yet unstudied for problems with non-classical viscous damping. (In Fig. 17, ρ and \bar{T} imply the spectral radius [6,20,48-50] and the oscillations period, respectively, and the difference of spectral radius from one at large value of $\Delta t/\bar{T}$ represents the capability of eliminating the oscillations with period \overline{T} in analysis with step Δt .) Despite the latter, there are experiences in the literature (e.g. see [51]), for extending the discussions on classical viscous damping to non-classical viscous damping, using complex variables [52], such that the classical case can be considered as a special case of the whole discussion. Therefore, it is reasonable to consider significant difference between the numerical damping of average acceleration and Wilson- θ methods, when the viscous damping is non-classical, as well. This explains the considerable difference between the base shears in Figs. 16(a) and 16(b). The numerical details however cannot be presented, because the figure corresponding to Fig. 17 is yet unavailable for problems with non-classical viscous damping (the case in this example). Only, as a simple rough study, the mid-parts of the base shears in Figs. 16(b) and 16(c) are compared in Fig. 18. Accordingly, the change of the base shear (versus time),

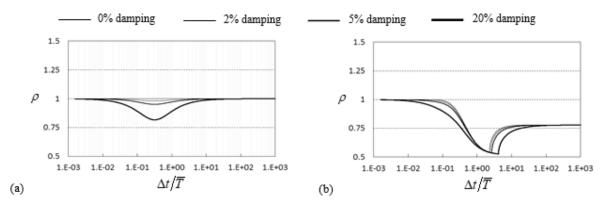


Fig. 17. Changes of spectral radius (ρ) with respect to $\Delta t/\bar{T}$ for the (a) average acceleration method [33], (b) Wilson- θ (θ =1.4) [38,39]

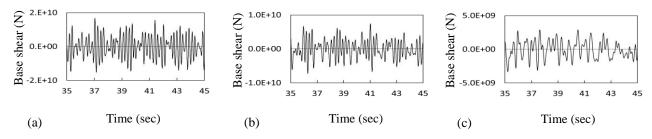


Fig. 18. A simple rough comparison between the base shears in Figs. 16(b) and 16(c): (a) Middle ten seconds of Fig. 16(c), (b) Middle ten seconds of Fig. 16(b) (the left figure), (c) Middle ten seconds of Fig. 16(b) (the right figure)

including the change of base shear increase (in Fig. 16(c)) to the base shear decrease (in Fig. 16(b)), occurs along with further removal of high frequency oscillations, in analysis with larger integration steps.

In answer to Question (b), if we compare the exact top displacement and the exact base shear in Fig. 16(c) (or even in Figs. 16(a) and 16(b)), the value of T is much larger for the top displacement. As a result, the value of $\Delta t/T$ and accordingly the value of $\Delta t/\bar{T}$ is much smaller for the top displacement, compared to the base shear. From the other side of view, because of the essentiality of convergence for time integration methods [3,4,6,11,12,20,21], the difference between the spectral radii of different time integration methods disappears for sufficiently small values of $\Delta t/\bar{T}$. (This is evident for problems with classical viscous damping in Fig. 17, as well as in the extended study considering many integration methods reported in [50].) Consequently, the effect of the numerical damping of Wilson- θ method on the top displacement is much less than the effect on the base shear. In other words, the integration has removed the high frequency oscillations from the response, and since the high frequency oscillations have negligible contribution to the top displacement (see Fig. 16(c)), the two top displacements in Fig. 16(b) are different negligibly. Due to a similar reason, the difference between the two base shears in Fig. 16(b) is noteworthy. This plus the role of T in Eq. (2) (though not rigorous) completes the answer to Question (b).

Finally, the reductions in analysis run-time, due to the SEB THAAT using Eq. (11), are equal to 29.32% and 22.34%, when using the Wilson- θ (θ = 1.4) and average acceleration methods, respectively. The difference of these values with 50% is because of the nonlinearity of the problem, explained previously in this paper.

Complementary discussion

In the previous sections, it was demonstrated that, in seismic response history analysis of mid-rise steel-structure buildings, by using Eq. (11), the SEB THAAT can reduce the analysis run-time, leading to sufficiently accurate responses, without prior knowledge about the response. Besides, in practice, buildings are subjected not only to ground motions, but also to gravity loads. As a result, the effect of using the SEB THAAT and Eq. (11) on the real analyses is even better than that discussed in the previous sections. This enhances the importance of the simplicity obtained from Eq. (11). Further discussion on some ambiguities and limitations is however essential.

The first ambiguity is whether the good performance observed in the presented examples relates to the seismic design code. In other words, is the observed good performance limited to the analysis of structures designed using the Iranian seismic standards? By using different design codes, the results of response history analysis with/without using the SEB THAAT will change. The simplicity and good performance of using the SEB THAAT and Eq. (11) will however not change, because of two main reasons. First, the scientific bases of seismic codes are close, and hence for a similar seismicity, the designs obtained from using different codes generally differ slightly. As a result, the effect of the change of the code on the performance of the SEB THAAT taking into account Eq. (11) would be reasonably tolerable. The second reason is that the codes used in the presented examples were the standards of Iran, and the seismicity of Iran is higher than many other regions of the world [53]. Therefore, when changing the design codes, the resulting *T* in Eqs. (2), (3), and (5) will probably increase. This implies even more suitability for Eq. (11) and more accuracy for the computed responses, when using seismic codes different from the codes of Iran.

In view of the presented examples (see Figs. 5 and 11), the second ambiguity is whether the SEB THAAT's performance is acceptable when the structural system is irregular in height or plan. In two separate studies [54,55], attention was paid to the performance of the SEB THAAT when applied to analysis of buildings' structures with irregularity in height or plan. In both studies, Eq. (11) provided an appropriate selection for n. A similar observation was made in a slightly different study on the SEB THAAT [56], as well. Accordingly, applying the SEB THAAT to seismic response history analysis of mid-rise steel-structure buildings with

irregularity in height or plan may be successful, for values of n equal to or greater than two. Irregularities simultaneous in both height and plan are yet not tested for the performance of the SEB THAAT. Considering this and for the sake of brevity, none of the tests on irregular structures is reported here. Therefore, the claims in this paper are limited to mid-rise steel-structure buildings categorized regular by the seismic codes. This limitation is however practically unimportant, because, it is decades that buildings' designers mostly prefer to design the structures to be regular according to the seismic codes 19,20,21,22,23,24 ; see also [25,26,57].

In view of the presented discussions, it is notable to add that few successful tests are reported on taller buildings [58] and buildings with concrete structure [56], as well. Besides, in view of the presented discussions and examples, and the more studied examples, not reported here for the sake of brevity, no limitation seems existing on the time integration method. Further investigation is essential.

Considering the limitations addressed above, the social importance and large number of mid-rise buildings, the simplicity of Eq. (11), the everyday smaller values of digitization steps, the generally time-consuming nature of response history analyses, and the considerable reductions in run-time reported in the presented examples, the achievements are significant. Therefore, the future of the presented research is promising, at least until Eq. (11) can be replaced with a better comment or computational procedure (see also [10]).

Conclusion

The SEB THAAT is a technique for accelerating different analyses of structural systems. In this paper, an engineering comment for applying the SEB THAAT in seismic response history analysis of mid-rise steel-structure buildings without any details about the response is proposed. The engineering comment implies obtaining the step enlargement scale n, in application of the SEB THAAT to response history analysis of midrise steel-structure buildings, from Eq. (11). By implementation of this comment, without notable effects on the response accuracy:

- (a) The SEB THAAT can be applied to response history analysis of mid-rise steel-structure buildings, in a much simpler way (Step 6 in Fig. 2 is considerably simplified).
- (b) The analysis run-times can be significantly reduced (50% for linear analyses).

This is a significant achievement, which, as a secondary achievement, can encourage structural analysts of mid-rise steel-structure buildings to use response history analysis in real projects.

Limitations exist for the proposed comment. In addition to those implied in the expression "mid-rise steel-structure buildings", the most important limitation is that the building structure must be "regular" according to the seismic code. This is however not a severe limitation, in view of the results of some recent researches, and the current practice of buildings structural design. Besides, the accuracy of the obtained responses need to be interpreted considering the numerical damping of the integration method. Other minor limitations exist, as well.

The future of the proposed comment is promising, with attention to its simplicity and effectiveness, and the fact that digitization steps are in every day decrease. Finally, extending the application of the proposed comment from response history analysis of mid-rise steel-structure buildings to other classes of analyses, and improvement of this comment are two areas for further research.

¹⁹ BHRC (Building and Housing Research Centre), Standard No. 2800-05, Iranian Code of Practice for Seismic Resistant Design, Iran, 2007 (in Persian).

²⁰ INBR (Iranian National Building Regulations), Iranian National Building Code, Part 10-Steel Structures, Iran, 1993 (in Persian).

²¹ ICC (International Code Council), IBC - International Building Code, Club Hills, USA, 2003.

²² BCJ (Building Centre of Japan), Structural Provisions for Building Structures, Tokyo, Japan, 2001.

²³ EAK, Greek Code for Seismic Resistant Structures, Athens, Greece, 2000.

²⁴ NRCC (National Research Council Canada), National Building Code of Canada, Canada, 2005.

Acknowledgments

The authors are grateful for the reviewers' comments, which have caused significant improvements in the paper. The role of the editorial team in quickly reviewing the paper and editing it very well is also sincerely appreciated. Finally, the first author acknowledges the financial support of the International Institute of Earthquake Engineering and Seismology (IIEES) in relation to the Projects 7510, 7528, and 7537.

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Aram Soroushian, Doctor of Philosophy (PhD) in Civil Engineering, Assistant Professor (Iran, Tehran) - International Institute of Earthquake Engineering and Seismology (IIEES), Faculty member of Structural Engineering Research Centre, a.soroushian@iiees.ac.ir

Abdolreza S. Moghadam, Doctor of Philosophy (PhD) in Earthquake Engineering, Associate Professor (Iran, Tehran) - International Institute of Earthquake Engineering and Seismology (IIEES), Faculty member of Structural Engineering Research Centre, moghadam@iiees.ac.ir

Ahmad Sabzei, Doctor of Philosophy (PhD) in Earthquake Engineering (Iran, Tehran) - International Institute of Earthquake Engineering and Seismology (IIEES), Research Assistant of Structural Engineering Research Centre, ahmad.sabzei64@gmail.com

Saeed Amiri, Master of Science (MS) in Civil Engineering, PhD Candidate (Canada, QC, Montreal) - Department of Civil, Geological and Mining Engineering, Polytechnique Montreal, saeed.amiri@polymtl.ca Aram Saaed, Master of Science (MS) in Civil Engineering, PhD Candidate (Iran, Tehran) - Structural Engineering Research Centre, International Institute of Earthquake Engineering and Seismology (IIEES), a.saaed@iiees.ac.ir

Ali Yahyapour, Master of Science (MS) in Civil Engineering (Iran, Tehran) - Structural Engineering Research Centre, International Institute of Earthquake Engineering and Seismology (IIEES), Research Assistant of Structural Engineering Research Centre, aliyahyapour7@gmail.com