# STUDY OF ENERGY DISSIPATION IN A TWO-PHASE SOIL SYSTEM



Tigran Petrosyan<sup>1</sup><sup>(1)</sup>, Arestak Sarukhanyan<sup>1</sup><sup>(1)</sup>

<sup>1</sup>National University of Architecture and Construction of Armenia, Yerevan, RA

Abstract: The dependence of energy dissipation coefficient on filtration properties and on creep characteristics of soil skeleton is examined. The study was carried out on the basis of the general solution of the joint task of creep and consolidation theory for a twophase soil system obtained on the basis of the generalized model of bulk forces, taking into account the interaction of soil phases, changes over time in the general stress state at any point of the soil and additional pressures in pore water, and incomplete transfer of external pressure to the pore water. Interactions between phases are taken into account in the solution of the one-dimensional joint task of the theory of creep and consolidation, and the creep kernel is taken in the form of an exponential function. The formula for the deformation of twophase soils is presented as the sum of two syllables due to primary and secondary consolidation of the soil. The strain formula uses experimentally obtained values of filtration characteristics and creep parameters of the soil skeleton. Using the formulas obtained, hysteresis loops under sinusoidal stress changes can be plotted and the energy expended per deformation cycle and the dissipation coefficient for two-phase soil can be obtained.

*Keywords:* consolidation, creep, two-phase soil, hysteresis, dissipation.

Tigran Petrosyan\*

E-mail: tipetrosyan@mail.ru

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## Introduction

It is known that under repeated stresses of a material there is a phenomenon of hysteresis, which is determined by energy dissipation. An immense number of works are dedicated to the study of the hysteresis phenomenon [8-11]. The mechanisms generating hysteresis are extremely diverse for different materials and test conditions, although in any case they are determined by rheological processes.

In many works [4-7], when studying the dissipation of mechanical energy and damping of natural vibrations, an attempt is made to find a relationship between energy dissipation and the parameters of material characteristics. The work [4] shows that the energy dissipation in the ground from cycle to cycle changes significantly, and the loading parameters strongly affect the value of the absorption coefficient. In this work [5], based on the data on the creep of the ground when using the theory of aging and heredity, hysteresis loops were drawn and compared with the experimental data for low-cycle creep, and it was found on the example of soils that the theory of heredity, in general, can be recommended for describing the absorption of energy. In this work [6], the dependence of the absorption coefficient on the period of cyclic loading, the degree of cycle asymmetry and on the cycle number for a material deforming according to the linear heredity theory is analyzed. The reviewed works do not take into account the structural peculiarities of materials with which the formation of the deformation and dissipative properties of materials is conditioned. Materials with pronounced structural features can be considered to be primarily two-phase (water-saturated) soils whose compaction under load is due to the process of water filtration from the soil pores and simultaneously to the creep of the soil skeleton [12,13].

In this work [14-17], the tasks of determining pore pressure in water-saturated soil under cyclic loading under conditions of absence of drainage were considered. Such a formulation and solution of the tasks was due to the fact that under short-term cyclic impacts pore pressure does not have time to dissipate and essentially the consolidation process does not begin. At the same time, it is obvious that there are conditions when cyclic

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impacts may be prolonged and at small filtration coefficients the consolidation process may develop over time. In this research [20] the one-dimensional task of consolidation of a water-saturated soil layer under an additional cyclic load was considered, assuming that the pore water contains air, in dissolved form and as bubbles, and the soil skeleton has rheological properties described by the equation of the modified Maxwell model.

What are the features of the consolidation process of water-saturated soil under alternating loads?

When the load is removed or, similarly, when additional negative load is applied, the process of swellingreconsolidation of water-saturated soil occurs. However, in the case of incomplete compaction-consolidation process, even with complete load removal, in a part of the soil body as a result of the swelling process development, areas of additional soil compaction temporarily arise and develop. Thus, with instant total removal from the consolidating soil layer, compressive stresses remain in the soil skeleton and excessive negative pressures in the pore water occur. At the permeable boundaries of the layer, the pressure is equal to zero, and the reconsolidation-swelling process gradually develops in the nearest zones of the layer, causing water inflow into the pores from the boundary drains and, what is especially important, from the middle part of the layer. As a result, in the middle part of such an incompletely consolidated layer, compressive stresses increase and, consequently, excessive negative pressures in pore water increase, i.e. reconsolidation of this part of the soil occurs. Over time, the reconsolidation zone decreases, and then only swelling of the ground occurs in the entire thickness, while the skeleton stress and water pressure gradually decrease to 0 [19]. Thus, if the soil sample is loaded and unloaded again, the above entire process will be repeated. And if the watersaturated soil, which constitutes a two-phase system, is subjected to periodic loading and unloading, the phenomenon of hysteresis appears, which causes a certain loss of mechanical energy.

The purpose of this work is to study the hysteresis effect in water-saturated (two-phase) soils under conditions of multiple compression combined with full unloading, on the basis of a one-dimensional task of filtration compaction (consolidation) of soils, taking into account the creep of the soil skeleton.

#### **Methods and Materials**

The study of hysteresis energy losses in water-saturated (two-phase) soils under periodic load changes was based on the general solution of the joint task of creep and consolidation theory for a two-phase soil system derived from the generalized bulk force model, taking into account the interaction of soil phases, time changes in the general stress state at any point of the soil and additional pressures in pore water, and incomplete transfer of external pressure to pore water [12,13].

This solution for the one-dimensional task is written down in the form [12]:

$$S(t) = hm_{\nu}p \left[ 1 + \int_{0}^{t} K(t - t_{0})dt_{0} - \frac{8}{\pi^{2}} \sum_{m=1,3,\dots}^{m=\infty} \frac{1}{m^{2}} \psi(t) \right],$$
(1)

where  $\psi(t)$  is a creep function.

Taking the creep kernel in the form [13]:

$$\overline{K}(t-t_0) = \delta e^{-\delta_1(t-t_0)}.$$
(2)

The creep function  $\psi(t)$ , taking into account the interaction of soil phases, is defined by the expression [12]:

$$\psi(t) = e^{-\left(\frac{\pi m}{2h}\right)^2 c_v t} + \frac{\delta}{\delta_1} \frac{e^{-\left(\frac{\pi m}{2h}\right)^2 c_v t} - e^{-\delta_1 t}}{1 - \left(\frac{\pi m}{2h}\right)^2 \frac{c_v}{\delta_1}},$$
(3)

where  $c_v = k_{\phi} / m_v \gamma_w$  is the consolidation coefficient of the two-phase soil, 2h is the thickness of the soil layer under bilateral drainage,  $\delta$  and  $\delta_1$  are the creep parameters.

As suggested in the work [12], the formula for the settlement of two-phase soils can be represented as:

$$S(t) = ph(m_{\nu}^{\rm I} U_0^{\rm I} + m_{\nu}^{\rm II} U_0^{\rm II}), \qquad (4)$$

where  $m_{\nu}^{I}$  and  $m_{\nu}^{II}$  are coefficients of primary and secondary soil consolidation,  $U_{0}^{I}$  and  $U_{0}^{II}$  are degrees of primary and secondary consolidation, respectively. The degree of primary consolidation  $U_{0}^{I}$  is determined by the expression:

$$U_0^{\rm I} = 1 - \frac{8}{\pi^2} \sum_{m=1,3,\dots}^{m=\infty} \frac{1}{m^2} e^{-\left(\frac{\pi m}{2\hbar}\right)^2 c_v t} .$$
 (5)

The degree of secondary consolidation  $U_0^{\text{II}}$  – by the formula:

$$U_0^{\text{II}} = 1 - e^{-\delta_1 t} - \frac{8}{\pi^2} \sum_{m=1,3,\dots}^{m=\infty} \frac{1}{m^2} \left[ \frac{e^{-\left(\frac{\pi m}{2h}\right)^2 c_v t} - e^{-\delta_1 t}}{1 - \left(\frac{\pi m}{2h}\right)^2 \frac{c_v}{\delta_1}} \right].$$
 (6)

Taking into account that the total ground settlement is defined by the formula:

$$S_{\infty} = h m_{\nu}^{\mathrm{I}} p \left( 1 + \frac{\delta}{\delta_1} \right). \tag{7}$$

And taking into account formula (4), we get the definition of the total degree of consolidation (due to filtration consolidation and simultaneously the creep of the soil skeleton) in the form of [12]:

$$U_0^{\Sigma} = \frac{S(t)}{S_{\infty}} = \frac{ph(m_v^{\rm I} U_0^{\rm I} + m_v^{\rm I} U_0^{\rm I})}{hm_v^{\rm I} p(1 + \frac{\delta}{\delta_1})}.$$
(8)

And accepting that

$$m_{\nu}^{\rm II} = m_{\nu}^{\rm I} \left(\frac{\delta}{\delta_1}\right). \tag{9}$$

We get

$$U_0^{\Sigma} = \left( U_0^{\mathrm{I}} + \frac{\delta}{\delta_1} U_0^{\mathrm{II}} \right) / \left( 1 + \frac{\delta}{\delta_1} \right). \tag{10}$$

For values of total consolidation degree  $U_0^{\Sigma} > 0.2$  in expressions (5) and (6) (for primary and secondary degrees of consolidation) we may limit it to the first term of the series.

Then the expression for the deformation of two-phase soil depending on time, in conditions of onedimensional task, with a uniform distribution of compaction pressures, taking into account the interaction of phases and skeletal creep, we get in the form of:

$$\varepsilon(t) = m_{\nu}\sigma(t) \frac{1 - \frac{8}{\pi^2} e^{-Mt} + \frac{\delta}{\delta_1} \left\{ 1 - e^{-\delta_1 t} - \frac{8}{\pi^2} \left[ \frac{e^{-Mt} - e^{-\delta_1 t}}{1 - M(c_{\nu}/\delta_1)} \right] \right\}}{1 + \frac{\delta}{\delta_1}},$$
(11)

where  $M = \frac{\pi c_v}{4h^2}$  and  $c_v = \frac{k_\phi}{m_v \gamma_w}$ , and  $\sigma(t)$  is the external load varying in time.

Consider the action of cyclic loading

$$\sigma(t) = \sigma_0[\sin(\omega t + \varphi_0) + \lambda], \qquad (12)$$

where  $\omega$  is the cyclic frequency,  $\varphi_0$  is the initial phase,  $\lambda$  is a constant determining the degree of asymmetry of cyclic loading.

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To determine the hysteresis loop area, which represents the energy  $\Delta W(n)$ , dissipated in one cycle of deformation, the formula [6] is used:

$$\Delta W(n) = \int_{T_n}^{T(n+1)} \sigma(t) \frac{\partial \varepsilon(t)}{\partial t} dt.$$
(13)

And the total mechanical energy W(n) spent in one cycle of deformation is determined by the formula [6].

$$W(n) = \int_{T_n}^{T\left(n+\frac{1}{2}\right)} \sigma(t) \frac{\partial \varepsilon(t)}{\partial t} dt , \qquad (14)$$

where  $T = \frac{2\pi}{\omega}$  is the cycle period, n is the cycle number.

#### **Results and Discussion**

The Substituting expressions (11) and (12) in formula (13) to determine the hysteresis loop area (the energy  $\Delta W(n)$  dissipated in one cycle of deformation) we will get:

$$\Delta W(n) = \frac{m_{\nu}\sigma_{0}^{2}}{1+\frac{\delta}{\delta_{1}}} \left\{ e^{-MTn} (1-e^{-MT}) \left[ \left( \frac{\lambda\omega^{2}}{M^{2}+\omega^{2}} + \frac{\lambda M^{2}}{M^{2}+\omega^{2}} - \frac{\omega^{2}}{M^{2}+4\omega^{2}} - \frac{M}{2} - \frac{M^{2}}{2(M^{2}+4\omega^{2})} \right) \left( -\frac{8}{\pi^{2}} - \frac{8\delta}{\pi^{2}} - \frac{\delta^{2}}{\delta_{1}^{2}+\omega^{2}} + \frac{\delta_{1}^{2}}{\delta_{1}^{2}+\omega^{2}} - \frac{\omega^{2}}{\delta_{1}^{2}+4\omega^{2}} + \frac{\delta_{1}}{2} - \frac{\delta_{1}^{2}}{2(\delta_{1}-4\omega^{2})} \right) \left( \frac{\delta}{\delta_{1}} - \frac{8\delta}{\pi\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \right) + \left( \frac{\lambda}{M} - \frac{M}{M^{2}+\omega^{2}} \right) \left( \frac{8\lambda M}{\pi^{2}} + \frac{8\delta\lambda M}{\pi^{2}\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \right) \right] + e^{-\delta_{1}Tn} \left( 1 - e^{-\delta_{1}T} \right) \left( \frac{\lambda}{\delta_{1}} - \frac{\delta_{1}}{\delta_{1}^{2}+\omega^{2}} \right) \left( \delta\lambda - \frac{8\delta\lambda}{\pi^{2}(1-M(c_{\nu}/\delta_{1}))} \right) \right\}$$
(15)

And the expression for the total mechanical energy W(n) expended for one cycle of deformation is obtained by substituting expressions (11) and (12) into formula (14):

$$W(n) = \frac{m_{\nu}\sigma_{0}^{2}}{1+\frac{\delta}{\delta_{1}}} \left\{ \frac{2\lambda}{\omega} \left(1+\frac{\delta}{\delta_{1}}\right) - \frac{8}{\pi^{2}} \left[ \lambda e^{-MTn} \left(1+e^{-\frac{MT}{2}}\right) - \frac{M^{2}+3\omega^{2}}{M^{2}+4\omega^{2}} e^{-MTn} \left(1-e^{-\frac{MT}{2}}\right) \right] + \frac{8\lambda M}{\pi^{2}} \left[ \frac{\lambda}{M} e^{-MTn} \left(1-e^{-\frac{MT}{2}}\right) \right] - \frac{\delta}{\delta_{1}} \left[ \lambda e^{-\delta_{1}Tn} \left(1-e^{-\frac{\delta_{1}T}{2}}\right) - \frac{\delta_{1}^{2}+3\omega^{2}}{\delta_{1}^{2}+4\omega^{2}} e^{-\delta_{1}Tn} \left(1+e^{\frac{\delta_{1}T}{2}}\right) \right] + \delta\lambda \left[ \frac{\lambda}{\delta_{1}} e^{-\delta_{1}Tn} \left(1-e^{-\frac{\delta_{1}T}{2}}\right) - \frac{\delta_{1}}{\delta_{1}^{2}+\omega^{2}} e^{-\delta_{1}Tn} \left(1+e^{\frac{\delta_{1}T}{2}}\right) \right] - \frac{8\delta}{\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \left[ \lambda e^{-MTn} \left(1+e^{\frac{MT}{2}}\right) - \frac{M^{2}+3\omega^{2}}{M^{2}+4\omega^{2}} e^{-MTn} \left(1-e^{-\frac{MT}{2}}\right) \right] + \frac{8\delta\lambda M}{\pi\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \left[ \frac{\lambda}{M} e^{-MTn} \left(1-e^{-\frac{MT}{2}}\right) - \frac{M}{M^{2}+\omega^{2}} e^{-MTn} \left(1+e^{\frac{\delta_{1}T}{2}}\right) \right] + \frac{8\delta\lambda M}{\pi\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \left[ \frac{\lambda}{M} e^{-MTn} \left(1-e^{-\frac{MT}{2}}\right) - \frac{M}{M^{2}+\omega^{2}} e^{-MTn} \left(1+e^{\frac{\delta_{1}T}{2}}\right) \right] + \frac{8\delta\lambda M}{\pi\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \left[ \frac{\lambda}{M} e^{-MTn} \left(1-e^{-\frac{\delta_{1}T}{2}}\right) - \frac{\delta_{1}^{2}+3\omega^{2}}{M^{2}(1-M(c_{\nu}/\delta_{1}))} \right] \right] + \frac{\delta\lambda}{\pi\delta_{1}(1-M(c_{\nu}/\delta_{1}))} \left[ \lambda e^{-\delta_{1}Tn} \left(1+e^{\frac{\delta_{1}T}{2}}\right) - \frac{\delta_{1}^{2}+3\omega^{2}}{\delta_{1}^{2}+4\omega^{2}} e^{\delta_{1}Tn} \left(1-e^{-\frac{\delta_{1}T}{2}}\right) \right] - \frac{\delta\lambda}{\pi^{2}(1-M(c_{\nu}/\delta_{1}))}} \left[ \frac{\lambda}{\delta_{1}} e^{-\delta_{1}Tn} \left(1-e^{-\frac{\delta_{1}T}{2}}\right) \right] \right] \right]$$

$$(16)$$

The absorption coefficient  $\Psi(n)$  is determined by the formula [7, 21]:

$$\Psi(n) = \frac{\Delta W(n)}{W(n)}.$$
(17)

Applying in expressions (15) and (16) experimentally obtained parameters of filtration and deformation (skeleton creep parameters) of soil, we can obtain the spent energy  $\Delta W(n)$  for one cycle of deformation and dissipation factor  $\Psi(n)$  for two-phase (water-saturated) soil.

### Conclusion

Thus, on the basis of this solution we will be able to:

1. Examine the dependence of dissipative properties of two-phase (water-saturated) soil on its filtration and deformation characteristics.

2. Control and regulate the dissipative properties in process of interaction and relationship of watersaturated soil phases.

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**Tigran Petrosyan, Doctor of Philosophy (PhD) in Engineering** (RA, Yerevan) - National University of Architecture and Construction of Armenia, Assistant at the Chair of Water Systems, Hydraulic Engineering and Hydropower, tipetrosyan@mail.ru

Arestak Sarukhanyan, Doctor of Science (Engineering), Professor (RA, Yerevan) - National University of Architecture and Construction of Armenia, Head of the Chair of Water Systems, Hydraulic Engineering and Hydropower, asarukhanyan51@mail.ru